

THE CONCEPTION OF BLOCKING POWER AS A KEY TO THE UNDERSTANDING OF THE HISTORY OF DESIGNING VOTING SYSTEMS FOR THE EU COUNCIL¹

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Abstract: *Unlike the classical approach to voting power, the approach presented in this paper makes a distinction between a voter's winning and blocking power and relates the latter kind of power to the number of small-size minimal blocking coalitions the voter can form with other voters. It is shown that the concept of blocking sheds light on the designing of voting systems for EU Council of Ministers from the very beginning to the Lisbon treaty.*

Key words: *voting game, voting power, blocking coalition, blocking power, EU Council.*

KONCEPCJA SIŁY BLOKOWANIA JAKO KLUCZ DO ZROZUMIENIA HISTORII PROJEKTOWANIA SYSTEMÓW GŁOSOWANIA DLA RADY UE

Streszczenie: *Przedstawione przez autora teoretyczne ujęcie siły głosu uczestnika zgromadzenia podejmującego decyzje przez głosowanie, odmienne od ujęcia klasycznego, opiera się na odróżnieniu „siły wygrywania” od „siły blokowania”, przy czym tę ostatnią dla każdego decydenta określa się jako zależną od liczby minimalnych koalicji blokujących małych rozmiarów z jego udziałem. W artykule pokazano, że koncepcja siły blokowania rzuca światło*

¹ The approach that is expounded here in Chapters 3 and 4 was for the first time proposed by the author on the '2nd Polish Symposium on Econo- and Sociophysics' (Kraków, April 21–22, 2006). His approach, supplemented with an analysis of blocking power as a relational concept, was presented subsequently at the International Workshop 'Distribution of Power and Voting Procedures in the European Union' (European Center Natolin, Warsaw, October 12–13, 2007). Some results given in this paper appeared earlier in a less technical form in two papers the author published in Polish (Sozański 2007a,b) and in his chapter (Sozański 2010) in the proceedings of the Natolin workshop. The author acknowledges his indebtedness to two anonymous reviewers whose remarks helped him significantly improve the text of the present paper.

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na historię projektowania systemów głosowania dla Rady Ministrów UE, od początków do traktatu lizbońskiego.

Słowa kluczowe: gra głosowania, siła głosu, koalicja blokująca, siła blokowania, Rada UE.

1. INTRODUCTION, OR ON POLITICIANS AND MATHEMATICIANS

Voting systems which have been invented and implemented throughout the history of political institutions have often come into being as a product of negotiations of *political actors*. It is quite natural for any member of an *assembly* that makes collective decisions by *voting* to demand that the *voting system* he would like to accept should be designed so as to guarantee to him as great *voting power* as possible. However, it is hardly ever clear, for the political actors themselves as well as for the observers of their disputes, what is the quantity each player wants to maximize. To define voting power operationally, the politicians need a theory, no matter whether they choose to theorize themselves or seek help from mathematicians or mathematical political scientists.

Mathematicians study *structural* properties of *mathematical objects* they call *voting games*. In particular, they analyze those voting games which are *mathematical models* of voting systems constructed by the politicians. To obtain such a model one must translate *voting rules* from the *legal* language into the *set-theoretic* formal language of mathematics. The example given below shows that the politicians happen to construct voting systems having rather complex structures.

The *legal definition* of the voting system for the EU council given in the Lisbon treaty

Decision rules are stated so as to make them applicable under varying number of EU members.

Article 16

4. As from 1 November 2014, qualified majority shall be defined as at least 55% of the members of the Council, comprising at least fifteen of them and representing Member States comprising at least 65% of the population of the Union.

A blocking minority must include at least four Council members, failing which the qualified majority shall be deemed attained.

A *mathematical model* of this voting system implemented for EU-27

Integer weights are used to enable exact calculations of the number of coalitions of particular types

Voting game $(H_1 \cap H_2) \cup H_3$

where H_1 , H_2 and H_3 are three *weighted voting games* with the same set of voters $\{1, \dots, 27\}$.

H_1 is a weighted voting game with relative population weights which add up to 1000 and quota $q_1=650$.

H_2 and H_3 are two 1 voter–1 vote games with quotas $q_2=15$ and $q_3=24$.

The mathematicians have defined various *indices of voting power*, often with the intention to provide the world of politics with some tools for practical use. However, the main “scientific” way of measuring voting power has so far held little appeal for the politicians. The latter used to rely on “naive theorizing,” sometimes combined with certain calculations, which – this is what we are going to show in this paper – become understandable if the practice of designing voting systems is interpreted in terms of a way of theorizing based on the concept of a small-size minimal *blocking coalition*.

2. THREE WAYS OF THEORIZING ON VOTING POWER

2.1. Naive theorizing

The scope of this very popular way of theorizing is limited to voting systems defined by assigning *weights* (number of nominal votes, share of the total population, etc.) to *voters* and setting a *quota* q , or the threshold that must be attained by the *total weight* of a set of voters in order that the set become a *qualified majority* entitled to pass any bill.

Under the naive approach, the *power* of a voter is equated with the voter’s weight or relative weight. Then, the power distribution does not depend on the choice of quota and the role of the latter reduces to determining the range of qualified majorities, which has to do with *efficiency* of the voting system.

2.2. Classical mathematical approach to voting power

The classical conception of voting power had its origin in few seminal papers (Penrose 1946, Shapley and Shubik 1954, Banzhaf 1965, and Coleman 1971) in which the key concept is critical membership in a winning coalition.

Let N denote the set of *voters* (actors/players) and \mathcal{W} the set of all winning coalitions (the term *coalition* is referred to any subset of N). To simplify notation, let $N = \{1, \dots, n\}$. A subset C of N is a *winning coalition* if the support of all members of C suffices – by virtue of certain “voting rules” – to pass any bill.

Definition 1. Voter i is a *critical* (synonymic terms: decisive/pivotal/swing) member of a winning coalition C ($C \in \mathcal{W}$) if i is a member of C ($i \in C$), and $C - \{i\}$ is not a winning coalition, that is, $C - \{i\} \notin \mathcal{W}$.

That is, if a member i of a winning coalition C fails to vote for a bill, the votes of the remaining members of C will no longer suffice for passing it. Under the Penrose-Banzhaf approach, the *voting power* of an actor i is directly proportional to the *number* $w_c(i)$ of *winning coalitions containing voter i as a critical member*.

The number $w(i)$ of *all* winning coalitions containing i can also be applied as a measure of voting power because it is *linearly related* to $w_c(i)$ through the following formula discovered by Dubey and Shapley (1979)

$$w(i) = \frac{1}{2}(w_c(i) + w)$$

where $w = |\mathcal{W}|$ is the number of all winning coalitions (the number of elements of a finite set Z will be noted $|Z|$).³

The most widely used normalized coefficient of voting power is the *Banzhaf index* $\beta(i)$. It is obtained by dividing $w_c(i)$ by the sum of $w_c(j)$ over $j=1, \dots, n$. The Banzhaf index is a measure of *relative* power, which means that its values over the set of voters add up to 1.

2.3. Mathematical formalization of the idea of blocking

The third approach shares all basic concepts with the classical approach, yet its most fundamental term is *blocking coalition* (“blocking minority” in EU documents)

Definition 2. C is a *blocking coalition* if: (i) $N-C$ is not winning ($N-C \notin \mathcal{W}$) and (ii) C is not winning ($C \notin \mathcal{W}$). That is, neither non-members nor members of C form a winning coalition.

Condition (i) means that C is given the power to prevent any bill from being passed. If all members of C refuse to vote for a bill, then it will not be passed, even if all remaining voters (members of $N-C$) vote for it. Since condition (i) is also satisfied by all winning coalitions (provided that C and $N-C$ cannot both be winning, which is a natural requirement), condition (ii) must be added in order to distinguish between winning and blocking coalitions. Condition (ii) implies that if all members of C vote for a bill, but all other voters fail to vote for it, then the bill will not be passed. Blocking coalitions are less powerful than winning coalitions; the latter can both block initiatives of non-members and push through their own initiatives.

Our definition of a blocking coalition brings back to life the original meaning given to this term by Lloyd Shapley (1962). “That sense – say Felsenthal and Machover (1998, p. 23) – agrees with common political parlance, in which the term is used to refer to a coalition that is able to stop a bill being passed but cannot force one through. However, subsequent usage in the voting-power literature has shifted to the broader sense of blocking, which we adopt here.”

3 To prove the formula, consider the sets $\mathcal{W}(i) = \{C \in \mathcal{W} : i \in C\}$, $\mathcal{W}_c^*(i) = \{C \in \mathcal{W}(i) : C - \{i\} \notin \mathcal{W}\}$, $\mathcal{W}^*(i) = \{C \in \mathcal{W} : i \notin C\}$, $\mathcal{W}_c^*(i) = \{C \in \mathcal{W}(i) : C - \{i\} \in \mathcal{W}\}$. Since the assignment $C \rightarrow C \cup \{i\}$ is a 1-1 mapping of $\mathcal{W}^*(i)$ onto $\mathcal{W}_c^*(i)$, we get $w^*(i) = w_c^*(i)$, which equation, together with $w = w(i) + w^*(i)$ and $w(i) = w_c(i) + w_c^*(i)$, yields the Dubey-Shapley formula.

This “broader sense”, which actually prevails in the literature, is obtained by defining a blocking coalition as any subset C of N such that $N-C$ is not winning, that is, by condition (i) only. The conception of blocking power that I’m going to develop later in this paper breaks with this tradition and builds on the following three general heuristic principles:

- blocking power should be distinguished from winning power;
- blocking power should be measured with the use of blocking coalitions;
- blocking coalition should be defined in theory in agreement with political practice.

Classical approach, which does not distinguish between two varieties of voting power, offers the ratio $w_c(i)/w$ as a measure of “preventive power.” This coefficient, defined by James Coleman (1971), however based on counting winning coalitions, has in fact to do with blocking power because it assumes the maximum value of 1 for a voter i if and only if i is a *vetoer* (that is, by definition, $\{i\}$ is a blocking coalition) or a *dictator* (that is, $\{i\} \in \mathcal{W}$). We see in dictatorship and the right of veto the extreme cases of winning and blocking power, respectively. While there must be only one dictator, maximal blocking power can be granted to all members of an assembly, as is the case with the *consensus game* having only one winning coalition made up of all players.

2.4. Abstract voting games

In the interest of nonmathematical readers who usually abhor too abstract discourse, until now I have not yet explicitly distinguished between two terms: “winning coalition” and “qualified majority,” the latter term being used by those who define decision rules in the language of law. However, not all historically known voting systems, including the one defined by the Lisbon treaty, have been designed solely by assigning weights to voters and setting a quota. Therefore, for the sake of generality, I must introduce now the theory of voting games as an *axiomatic mathematical theory*.

An abstract *voting game* is a *mathematical object* of the form (N, \mathcal{W}) , where N is a finite set of *voters*, and \mathcal{W} is a collection of subsets of N called *winning coalitions*. Note that the set N , called the *assembly* of voters (N is the *base set* of the *mathematical object* (N, \mathcal{W}) with *structure* \mathcal{W}), is also referred to as a coalition (a subset of N), in which case it is termed the *grand coalition*.

The starting point for building the *mathematical theory of voting games* are not concrete *voting rules*, but abstract *axioms* assumed to be met by \mathcal{W} . The following axioms seem most convenient insofar as one would like to develop a formal theory in such a way that it could serve as a basis for typical political applications:

- A₁ $\mathcal{W} \neq \emptyset$ (there exists at least one winning coalition);
- A₂ If $C \in \mathcal{W}$ and $C \subset D \subset N$, then $D \in \mathcal{W}$ (any set of players D which contains a winning coalition is also a winning coalition; $C \subset D$ stands for ordinary inclusion, encompassing the case where $C=D$);
- A₃ If $C \in \mathcal{W}$, then $N-C \notin \mathcal{W}$ (the non-members of a winning coalition do not form a winning coalition).

The mathematicians love general concepts and general theorems. Thus, even those willing to attract nonmathematical readers (Felsenthal & Machover, 1998; Straffin, 1993) begin theory building from defining a *simple voting game* as a mathematical object which meets only two axioms A₁ and A₂ (A₁ is usually replaced by B₁: $N \in \mathcal{W}$, which under our axiomatics follows from A₁ and A₂). Axiom A₃ is used then to define a particular class of simple voting games, referred to as *proper* simple voting games. In this paper, for convenience, the term *voting game* will be used for this special case. This case corresponds to a widely accepted *political decision rule* according to which two *contradictory* bills, the one supported by C and the other supported by $N-C$, may not be passed simultaneously, which would be possible if Axiom A₃ did not hold.

A coalition will be called *losing* if its complement $N-C$ is winning. Note that in the classical strand of theorizing, the term “losing” is used synonymously with “not winning.” To avoid confusion, the readers of this paper who are familiar with voting game literature should keep in mind all the time the different meaning that is given throughout this paper to the term *losing*.

The definition of a losing coalition implies that, for any C , C is losing if and only if $N-C$ is winning. Hence $|\mathcal{W}| = |\mathcal{L}|$ where \mathcal{L} stands for the set of losing coalitions. The remaining subsets of N are blocking coalitions. Let \mathcal{B} denote their set. We have $2w+b=2^n$ where $b = |\mathcal{B}|$.

3. THE MEASUREMENT OF BLOCKING POWER

3.1. Can blocking power be measured by analogy with winning power?

Once $w(i)$ is a measure of winning power, can $b(i)$ – the number of blocking coalitions containing actor i – be used to construct an index of blocking power? The answer is negative due to the following formula

$$b(i) = 2^{n-1} - w$$

which holds true for all i . To prove it, notice that the mapping $C \rightarrow N-C$ of the set of all coalitions onto itself establishes a one-to-one correspondence between $\mathcal{B}(i) = \{C \in \mathcal{B} :$

$i \notin C\}$ and $\mathcal{B}^*(i) = \{C \in \mathcal{B}: i \notin C\}$, two disjoint sets which make up \mathcal{B} . Therefore, $b(i) = \frac{1}{2}b$, but $b = 2^n - 2w$.

Consider in turn the parameter $b_c(i) = |\mathcal{B}_c(i)|$ where $\mathcal{B}_c(i) = \{C \in \mathcal{B}(i): C - \{i\} \notin \mathcal{B}\}$ is the set of blocking coalitions containing voter i as a critical member, where “critical” means that if actor i leaves a blocking coalition C , then $C - \{i\}$ is not a blocking coalition (then it must be losing). We prove the following fact about $b_c(i)$:

$$b_c(i) \leq w_c(i), b_c(i) = w_c(i) \text{ if and only if, for any } C \in \mathcal{W}_c(i), C - \{i\} \in \mathcal{B}$$

Proof. Clearly, the mapping $C \rightarrow (N - C) \cup \{i\}$ assigns different sets to different subsets of N containing i . Hence, $b_c(i) \leq w_c(i)$ provided that $C \in \mathcal{B}_c(i)$ implies that $(N - C) \cup \{i\} \in \mathcal{W}_c(i)$. If $C \in \mathcal{B}_c(i)$, then $C - \{i\} \notin \mathcal{B}$ and consequently $C - \{i\} \in \mathcal{L}$, which implies that $N - (C - \{i\}) = (N - C) \cup \{i\} \in \mathcal{W}$, but $N - C \in \mathcal{B}$ because $C \in \mathcal{B}$, so that $(N - C) \cup \{i\} \in \mathcal{W}_c(i)$.

The above iff condition means that the defection of a coalition member can never result in a direct transition from winning to losing coalition. Our analysis of voting games designed for the EU Council has revealed that this condition is met by these games. Thus, the idea to use $b_c(i)$ to define the *Banzhaf-like index of blocking power* has turned out of little practical value, as blocking power would be equal to winning power for the games in question.

In general, the condition “if $C \in \mathcal{W}_c(i)$, then $C - \{i\} \in \mathcal{B}$, for any C ” need not always be met as shown by the following simple example. Let $N = \{1, 2, 3\}$ and \mathcal{W} consist of $C_0 = N$, $C_1 = \{1, 2\}$, and $C_2 = \{1, 3\}$. Their complements $C_3 = \emptyset$, $C_4 = \{3\}$, and $C_5 = \{2\}$ are losing coalitions, while $C_6 = \{1\}$ and $C_7 = \{2, 3\}$ (the remaining 2 out of $2^3 = 8$ subsets of N) are blocking coalitions. The defection of voter 1 from either of two coalitions C_1 and C_2 that form $\mathcal{W}_c(1)$ results in transforming these coalitions into losing coalitions C_5 and C_4 , respectively. Note that player 1 is a vetoer in this game, his *blocking power* thus being maximal, but he is not a dictator, even though his *winning power* exceeds that of other players, as 1 is a member of all winning coalitions. The idea to distinguish two kinds of voting power does mean the claim that they are independent of each other.

The condition implying that $b_c(i) = w_c(i)$ is not generally met also for a narrower class of abstract voting games, *weighted voting games*, or the games which are obtained each by assigning to any voters positive numbers p_1, \dots, p_n , called *weights*, setting a number q (*quota*) such that $\frac{1}{2}(p_1 + \dots + p_n) < q \leq p_1 + \dots + p_n$, and defining \mathcal{W} as the set of all subsets C of N such that the sum of weights over C equals at least q . The abstract 3-player game we have shown as a counter-example can be represented as a weighted voting game with $p_1 = 4$, $p_2 = p_3 = 3$ and $q = 7$.

3.2. Minimal winning and minimal blocking coalitions

Definition 3. A winning (blocking) coalition C is called *minimal* if no proper subset of C is winning (blocking). Formally, $C \in \mathcal{W}$ ($C \in \mathcal{B}$) is minimal if for any D such that $D \subset C$ and $D \neq \emptyset$, $D \notin \mathcal{W}$ ($D \notin \mathcal{B}$).

Equivalently, C is minimal if every member of C is critical, that is, C has no redundant members whose defection would not change the coalition type. We define in turn a collection of *structural parameters*⁴ based on counting minimal blocking coalitions. Analogous parameters $w_m, w_{m,k}, w_m(i), w_{m,k}(i)$ are defined with the use of minimal winning coalitions.

- \underline{b}_m – number of all minimal blocking coalitions
- $\underline{b}_{m,k}$ – number of all minimal blocking coalitions of size k
- $b_m(i)$ – number of minimal blocking coalitions containing voter i
- $b_{m,k}(i)$ – number of minimal blocking coalitions of size k containing voter i

The numbers \underline{b}_m and $\underline{b}_{m,k}$ ($k=1, \dots, n$) characterize a given game $G=(N, \mathcal{W})$ as a whole, so we will refer to them as *global* parameters. Parameters b_m and $b_{m,k}$ (notice the difference in notation between \underline{b}_m and b_m) are defined as mappings which assign values to particular players of G ; these mappings will be referred to as *local* parameters.

The calculation of all blocking parameters of either type may begin from determining the most elementary quantities: $b_{m,k}(i)$, for $k=1, \dots, n$ and $i=1, \dots, n$. Next we get $b_m(i)$ for any i as the sum of $b_{m,k}(i)$ over k ranging from 1 to n . The formula $k \cdot \underline{b}_{m,k} = \sum_i b_{m,k}(i)$ allows us to find in turn $\underline{b}_{m,k}$. At the last step we obtain \underline{b}_m as the sum of $\underline{b}_{m,k}$ over $k=1, \dots, n$.

If actor i 's blocking power were to be measured by the number $b_m(i)$ of all minimal blocking coalition containing i , one would obtain the following result for the weighted voting game used by the Council of Ministers in EU-15 ($q=62$). In this game, players 1 through 4 have the same weight $p_i=10$, and, consequently, they have the same value of b_m and of any other structural parameter.⁵

⁴ To formally define a *structural* parameter, we must first introduce the notion of isomorphism for voting games. Two voting games $G_1=(N, \mathcal{W}_1)$ and $G_2=(N, \mathcal{W}_2)$ with the same set of players N are said to be *isomorphic* through a 1-1 mapping α of N onto N if for any $C \subset N$, $C \in \mathcal{W}_1$ if and only if $\alpha(C) \in \mathcal{W}_2$, where $\alpha(C) = \{\alpha(i) : i \in C\}$. Every *permutation* α with this property is referred to as *isomorphism* of G_1 onto G_2 . *Automorphisms* of $G=(N, \mathcal{W})$ are isomorphisms of G onto G . Any mapping f assigning numerical values to the elements of N (such mappings will be termed *parameters of a player*) is called *structural* if $f(\alpha(i)) = f(i)$ for any $i \in N$ and for any automorphism α of G . *Parameters of a voting game* characterize a voting game as a whole. Those which assume the same value for isomorphic games will be called *structural*.

⁵ It is a consequence of structural interchangeability of the players having the same weight. Two players i and j in any voting game G are said to be *structurally interchangeable* if $j = \alpha(i)$ for some automorphism α of G (structural interchangeability is an *equivalence relation* on N : it is *reflexive*, *symmetric*, and *transitive*). Any structural parameter assumes the same value for any two structurally interchangeable players. In a weighted voting game G , any two players i and j such that $p_i = p_j$ are structurally interchangeable because the mapping τ of N onto N such $\tau(i) = j$, $\tau(j) = i$, and $\tau(h) = h$ for $h \neq i, j$ is an automorphism.

Table 1
Numbers of minimal winning and blocking coalitions in EU-15

ρ_i (weight)	$w_m(i)$	$b_m(i)$
$\rho_1=\rho_2=\rho_3=\rho_4=10$	674	324
$\rho_5=8$	619	334
$\rho_6=\rho_7=\rho_8=\rho_9=5$	542	489
$\rho_{10}=\rho_{11}=4$	511	494
$\rho_{12}=\rho_{13}=\rho_{14}=3$	485	485
$\rho_{15}=2$	375	375

While the ordering of 15 countries with respect to the values of w_m agrees with their ordering with respect to weights (this feature is special to this game, such consistency is not a property of all weighted voting games), the values of the second local parameter behave otherwise: small countries surpass large countries in the number of *all* minimal blocking coalitions. This is because they can form many such coalitions among themselves. However, since these coalitions must have many members, their formation may turn out more difficult than the formation of smaller size coalitions in which strong players ally with weaker players. Why did the largest members of the Fifteen agree on adopting a voting system which did not give them more blocking opportunities than to the smaller members? Apparently they did not care too much about their access to as many as possible minimal blocking coalitions regardless of their size. They rather wanted to be able to form as many as possible minimal blocking coalitions of as small as possible size.

3.3. Measuring blocking power with the use of small minimal blocking coalitions

The above reasoning leads to the idea that the blocking power of a voter should depend on the smallest size of a minimal blocking coalition the voter can form with other voters and on how many alternative *small* minimal blocking coalitions are available to him.

What size of a minimal blocking coalition, besides $k_{\min} = \min\{k: \underline{b}_{m,k} > 0\}$, should be considered small? Such a question should be asked the users of a given voting system. Every player i would probably agree that any minimal blocking coalition of the size $k_{\min}(i) = \min\{k: b_{m,k}(i) > 0\}$ ⁶ is small, as *for him* it is the smallest possible size.

If $w_m(i) = 0$, then $b_m(i) = 0$, which means that a player who is deprived of any winning power, doesn't have any blocking power, either. Clearly, the converse implication (if $b_m(i) = 0$, then $w_m(i) = 0$) is not true for any voting game such that $b = |\mathcal{B}| = 0$. If $b > 0$,

⁶ We put $k_{\min}(i) = 0$ if $b_m(i) = 0$, that is, if $b_{m,k}(i) = 0$ for any k .

it is not true, either.⁷ Thus, the case of the smallest blocking power and that of the smallest winning power do not necessarily coincide, which is another reason for which one needs to distinguish between two kinds of voting power.

There exist many voting games, where $b=0$, or $B=\emptyset$, that is, the players have no opportunity for blocking. In such games, which are called *strong*, one can consider only winning power. Strong games are most efficient, *efficiency* (named by Coleman the “power of a collectivity to act”) of any voting game being defined as the ratio of w to $2w+b$.

We define *small* minimal blocking coalitions as those of the size ranging from k_{\min} to the maximum, noted k_{\max} , of $k_{\min}(i)$ over all i .

Let us define in turn the simplest coefficient of blocking power as the *ratio of the number of small minimal blocking coalitions containing voter i to the number of all small minimal blocking coalitions*, symbolically

$$\gamma(i) = \frac{\sum_{k=k_{\min}}^{k_{\max}} b_{m,k}(i)}{\sum_{k=k_{\min}}^{k_{\max}} b_{m,k}}$$

This structural parameter, which is defined only if $b>0$ (otherwise the denominator is 0), disregards the size of small blocking coalitions. We leave for further analyses the problem of how to refine it, so that in assessing the amount of blocking power that the players have each in a voting game with $k_{\min} \neq k_{\max}$ one takes into account the distribution by size of small blocking coalitions containing a given voter i . It may be that one should resort to the same method which was used by Deegan and Packel (1979) to define their coefficient of winning power (based on counting *all* minimal winning coalitions). A key theoretical question is such: the access to how many minimal blocking coalitions of size $k+1$ (e.g. 4) counts for a player in estimating his total blocking power as much as his membership in *one* minimal blocking coalition of size k (e.g. 3). To answer this question, a theorist may need to consult potential theory users. In this paper, I am more interested in characterizing the shape of what will be called here the blocking structure of a voting game than in inventing new coefficients and examining their formal properties and/or their behavior in various games.

⁷ I owe to one of two anonymous reviewers the following counter-example. It is the game G with $N=\{1,2,3,4,5\}$ in which all coalitions C such that $|C|=3$ except $\{1,2,3\}$ and $\{3,4,5\}$ are the only minimal winning coalitions. It is not difficult to verify that the only minimal blocking coalitions in G are $\{1,2\}$ and $\{4,5\}$. Thus, $b_m(3)=0$, but $w_m(3)=4$.

4. THE BLOCKING STRUCTURE OF A VOTING GAME

4.1. Formal properties of the blocking structure

The constructors of voting systems for the EU Council seem to be little interested in methods of *quantifying* blocking power. They have always been more concerned with the *shape of blocking structure* with focus on its *lowest level*, or the distribution of the number of minimal blocking coalitions of the smallest size k_{\min} . The set of such coalitions of which the number is usually pretty small can often be determined by political users without the help of experts.

Definition 4. For any voting game with n players, the *blocking structure* is the sequence of sequences $(b_{m,k}(i): i=1,\dots,n)$ with k ranging from k_{\min} to k_{\max} .

In describing the shape of a blocking structure, one needs to take into account the following formal properties or parameters:

- the smallest size of a minimal blocking coalition; this parameter has always been considered important in designing voting games for the EU Council (k_{\min} was always equal to 2, 3 or 4 with the tendency to be raised with successive EU enlargements);
- the number of levels $(k_{\max} - k_{\min} + 1)$ in the blocking structure; in EU games it has never exceeded 3;
- the number of voters with $b_{m,k}(i) > 0$ at level k ; the set of players who take part in minimal blocking coalitions of the smallest size will be referred to as the *premier league*;
- even vs. uneven distribution of non-zero values $b_{m,k}(i)$ on each level;
- last but not least, *regularity* (to be defined below) or irregularity of the blocking structure.

4.2. Regularity of the blocking structure

To define regularity, we must first introduce the notion of “consistency” for two parameters (structural or not) f and g that assign numerical values to the players in a voting game $G=(N,\mathcal{W})$. We say that f and g are *consistent* with each other if there does not exist a pair of players $\{i, j\}$ such that $f_i > f_j$ and $g_i < g_j$ (f_h and g_h stand for the values of f and g for player h).

Definition 5. A voting game G is said to have a *regular blocking structure* if any two parameters $b_{m,k}$ and $b_{m,l}$ of a voter, where k and l lie in the range from k_{\min} to k_{\max} , are consistent with each other.

A one-level blocking structure ($k_{\min}=k_{\max}$) will also be treated as regular, although this property is defined here basically for voting games with multi-level blocking structure. If a voting game has a two-level blocking structure that is not regular, then the voters may find it troublesome to make inter-player comparisons with respect to the degree of blocking power. Indeed, it may be difficult for a voter who occupies a high position on one level of the blocking structure and low position on the other level to locate his place in the *overall* ordering of voters with respect to the size of blocking power or even to define this ordering itself.

For a weighted voting game, the definition of regularity is supplemented with the requirement that the assignment of weights to players be consistent with every component $b_{m,k}$ of the blocking structure.

To illustrate the use of the concepts we have introduced in the Sections 4.1 and 4.2 we close Chapter 4 with an analysis of the Nice “triple majority voting system” that was in use in EU-27. This game, which is a nontrivial example of a voting game with irregular blocking structure, is a little tricky to analyze because it is the intersection of 3 voting games.

The *intersection* of two voting games (N, \mathcal{W}_1) and (N, \mathcal{W}_2) over the same assembly N of voters is defined as a voting game of the form (N, \mathcal{W}) , where $\mathcal{W} = \mathcal{W}_1 \cap \mathcal{W}_2$ is the intersection of the sets \mathcal{W}_1 and \mathcal{W}_2 of winning coalitions of the two games. One can easily prove that the set \mathcal{L} of losing coalitions determined by \mathcal{W} equals $\mathcal{L}_1 \cap \mathcal{L}_2$. The formula for \mathcal{B} is more complicated: $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2 \cup (\mathcal{L}_1 \cap \mathcal{W}_2) \cup (\mathcal{W}_1 \cap \mathcal{L}_2)$ (its simple but somewhat tedious proof is omitted here). The formula implies that the use of intersection as a method for constructing new voting games may result in extending the range of blocking opportunities.

4.3. The blocking structure of the Nice voting game for EU-27

The Nice voting game for EU-27 (more exactly, its mathematical model constructed here upon the assumption that weights in the third component are integers that add up to 1000) has the form

$$G = G_1 \cap G_2 \cap G_3$$

where G_1 , G_2 , and G_3 are three weighted voting games described below.

G_1 is obtained by distributing 345 “nominal votes” among the members of EU-27 and setting the quota to 255;

G_2 is 1 voter–1 vote voting game with quota 14;

G_3 is a weighted voting game which corresponds to the third voting rule introduced in the Nice treaty by the following statement: *When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Member States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is not shown to have been met, the decision in question shall not be adopted.*

If weights are not fixed in advance as part of the definition of the game, as is the case for the third component of the Nice game, then the calculations and their results must depend on what *external* data is used to be taken as *input* for a given computer program the analyst needs to determine exact numbers of particular types of coalitions. The population data vary over time, in addition they are presented with varying degree of precision. Throughout this paper I use the Eurostat population data which were official input to decision procedures during the German presidency in the first half of 2007. The program (POWERIND⁸) I wrote in 2004 in Quick Basic 4.5 transforms the population data (see Table 2) into integer weights which add up to 1000, so that 620 becomes the integer counterpart of the relative quota of 62% that is given in the above clause quoted from the Nice treaty.

Formally, the Nice game G has a 3-level blocking structure, but the players except the weakest one (Malta) probably don't take into account the third level of minimal blocking fives in estimating the size of their blocking power.

The shape of the blocking structure of G is jointly determined by G_1 and G_3 . The 1 voter–1 vote game G_2 has no effect on the set of small minimal blocking coalitions of G .

For now, I don't know to what degree the set of *minimal blocking fives* depends on G_1 and G_3 . However, one can verify that: (1) every blocking five must contain at least 2 out of 6 largest states; (2) every 2 members of the Big Six can block any initiative of 4 remaining members with the help of 3 weaker players.

The examination of the set of minimal blocking threes and fours has led to the discovery of the following facts.

G has 4 blocking threes, all inherited from G_3 (the total integer population weight of each coalition is given in brackets): {Germany, France, UK}(417), {Germany, France, Italy}(414), {Germany, UK, Italy}(408), {Germany, France, Spain}(384).

⁸ Unfortunately, my program (available upon request from the author) admits no more than 27 players, so it cannot be used for EU-28 (in addition, it doesn't work under versions of Windows newer than XP).

Table 2
The blocking structure of the Nice game

EU-27	Nice wght	Population		$b_{m,k}(l)$		
		in 1000	wght	$k=3$	$k=4$	$k=5$
1. Germany	29	82438	167	4	90	651
2. France	29	62886	128	3	109	663
3. UK	29	60393	122	2	128	666
4. Italy	29	58752	119	2	125	678
5. Spain	27	43758	89	1	124	563
6. Poland	27	38157	77	0	136	590
7. Romania	14	21610	44	0	16	678
8. Netherlands	13	16334	33	0	16	528
9. Greece	12	11125	23	0	16	405
10. Portugal	12	10570	21	0	16	405
11. Belgium	12	10511	21	0	16	405
12. Czech R.	12	10251	21	0	16	405
13. Hungary	12	10077	20	0	16	405
14. Sweden	10	9048	18	0	16	239
15. Austria	10	8266	17	0	16	239
16. Bulgaria	10	7719	16	0	16	239
17. Denmark	7	5427	11	0	12	76
18. Slovakia	7	5389	11	0	12	76
19. Finland	7	5256	11	0	12	76
20. Ireland	7	4209	8	0	12	76
21. Lithuania	7	3403	7	0	12	76
22. Latvia	4	2295	5	0	2	86
23. Slovenia	4	2003	4	0	2	86
24. Estonia	4	1345	3	0	2	86
25. Cyprus	4	766	2	0	1	88
26. Luxembourg	4	460	1	0	1	88
27. Malta	3	404	1	0	0	72
	345	492852	1000	4	235	1729

All blocking threes contain Germany. Notice also that the total weight of {France, UK, Italy}, or the strongest three without Germany, equals 369, which helps us guess why the *relative quota* in G_3 was set to 62% rather than to 63% or to a higher value. Let me add that the governments and their experts must have been aware of the *political* meaning of setting the quota to 62% from the outset. In 2000 the total population of {France, UK, Italy} formed 36.3% of the total population of EU-27; since that year this quantity was slightly growing to reach the value of 37.1% in 2008.

The set of *minimal blocking fours* consists of 235 coalitions of which only 3 owe the property of blocking *solely* to the population game G_3 . These 3 blocking fours are obtained by appending to the coalition {Germany, UK, Spain} one of 3 very small states, Latvia, Slovenia or Estonia. The total weights of the 4-player coalitions are 383, 382 and 381, respectively, so these coalitions may no longer remain blocking if the EU population distribution changes.

The remaining 232 minimal blocking fours in the Nice game come (not always exclusively) from game G_1 defined with the use of fixed political weights. Thus, the hybrid blocking structure of the Nice game will remain *stable* (relatively insensitive to population changes) at least at the second level.

While Germany is the leader on the first level of blocking threes, it drops to the last position within the Big Six on the level of minimal blocking fours where Poland has unexpectedly taken the lead, however, by being compensated for its absence in blocking threes. On the level of minimal blocking fives, Poland and Spain do rather poorly, Italy and Romania are now ahead of all other players, which makes the blocking structure of the Nice game highly *irregular*.

5. RELATIONAL ANALYSIS OF BLOCKING POWER

Along with the *distributive* understanding of *power*, political scientists have always construed power as a *relational* concept, as illustrated by Dahl's definition (1957: 202-203): "A has power over B to the extent that he can get B to do something that B would not otherwise do." This idea can also be formalized in the context of our conception of blocking power.

Given the set S of all *small* minimal blocking coalitions in a voting game, we define four coefficients that are to render four different aspects of institutionally-based political relationship of two actors A and B. Let $S_{A \cup B}$ denote the set of coalitions in S containing player A or player B, and $S_{A \cap B}$ – the set of coalitions in S containing both A and B.

The ratio $I_{AB} = |S_{A \cup B}| / |S|$ measures *structural importance* of the pair {A,B} within the voting system. The ratio $C_{AB} = |S_{A \cap B}| / |S_{A \cup B}|$ is a measure of *system-forced potential cooperation* of A and B in blocking initiatives of other players. Both coefficients are symmetric, that is, $I_{AB} = I_{BA}$ and $C_{AB} = C_{BA}$.

Let S_{A-B} stand for the set of coalitions in S containing A but *not* B, or those coalitions A can use to block B's initiatives. The ratios $P_{AB} = |S_{A-B}| / |S_{A \cup B}|$ and P_{BA} (defined similarly) are measures of blocking power A and B have in relation to each other. If $P_{AB} > P_{BA}$, A is said to have *blocking power advantage* over B. Notice that a player *i* has blocking power advantage over player *j* if and only if $\gamma(i) > \gamma(j)$, that is, the order of players with respect to the values of the blocking power parameter determines their unequal opportunities to block each other's initiatives.

The concepts of which the definitions I decided to recall here after my earlier paper (2010) can be used to enrich the historical analyses (given in Chapter 7) with more results concerning the question of how the *relations* within the union of states evolved with each enlargement and each change of the voting system for the Council of Ministers. Further inquiry into of this issue, which is beyond the scope of the present study, is a task that remains yet to be done. In my earlier paper (2010, p. 88-89), I presented only one example of relational analysis. It was shown there that the relationship between France and Germany would change a lot after the replacement of the Nice voting system with the one established by the Lisbon treaty.

6. RELATIONSHIPS BETWEEN THREE APPROACHES TO VOTING POWER

6.1. Naïve theorizing vs. classical mathematical approach

Let p_i and p_j denote the weights of players i and j in a weighted voting game with quota q . If $p_i > p_j$, then $w(i) \geq w(j)$, which implies that $w_c(i) \geq w_c(j)$ and $\beta(i) \geq \beta(j)$. Thus, two parameters of a player, the assignment of weights and the Banzhaf index are always consistent with each other.⁹

Słomczyński and Życzkowski (2006) suggest to calculate a quota from the weights in such a way that the *relative weight* of each voter and the respective value of the Banzhaf index are approximately equal. They found a formula for such a quota for the case of weighted voting games with weights computed as square roots of original population weights. For the square root game designed by them for EU-27 the relative quota with this property equals 61.6%. If square root weights are represented in the form of integers which add up to 345, the absolute quota corresponding to .616 equals 213. The use of such a quota might help political users reconcile their naive approach to voting power with the classical approach. However, the success of this strategy of promoting the mainstream way of mathematical theorizing on voting power depends on whether the politicians agree to apply it in practice. A scholar can do nothing to gain acceptance for his or her approach when they hear from a politician: "I'm sorry, but your way of measuring voting power much differs from mine."

Nevertheless, the classical approach is not doomed to remain "academic." According to Dubey and Shapley (1979, p. 100): "The main ideas underlying the game-theoretic approach to power eventually found wide legal acceptance; indeed,

⁹ The assignment of weights to players need not be a structural parameter. For example, the weights $p_1=4$, $p_2=3$, $p_3=2$, with $q=5$ generate a voting game in which all 3 players are interchangeable with one another, but their weights differ among one another, so a radical naive theorist might think that the three actors are not equally powerful. The same voting game will be obtained by setting $p_1=p_2=p_3=1$ and $q=2$. These weights reflect status equality among three actors. This assignment of weights to them is now a structural parameter.

in New York State today, some of the county supervisorial boards are constituted according to a form of Banzhaf's index, in an attempt to equalize the representation of citizens living in municipalities of different size."

If a choice between two or more games has to be made by political actors who will subsequently play the chosen game, then an actor's evaluation of each considered game in terms of how strong position he gets in it depends on the power measure used. If a player's "strength" is quantified by two different measures, then their consistency does not suffice to find a *compromise solution* of the problem of which game to choose. The example given below provides a plausible explanation of why Germany refused to accept the *Jagiellonian game*, or the game with square root weights and the relative quota 61.6%.

Let us compare this game with two other voting games for EU-27, so differently evaluated by Poland and Germany, the Nice game and the Lisbon game. Under the naive approach the Nice "triple majority system" is usually identified with its main component, or the game G_1 with "politically agreed-on weights" (nominal votes) and quota 255.

If you compute the mean of the Banzhaf index values for the Nice game and the Lisbon game, you will get .0971, which exceeds by .014 the respective value for the Jagiellonian game. Thus, the latter game can in fact be regarded as a compromise solution. But if you rely on the naive approach, as probably did the German government, and take into account relative weight or absolute integer weight (calculated so as to imitate Nice weights which add up to 345), you will arrive at a completely different conclusion: the "Jagiellonian compromise" is by no means in the middle between two games considered best and worst for Germany. The values of the *Shapley-Shubik index* (the second most popular classical power coefficient, a special case of *Shapley value*), which in the three games are equal for Germany to .0874, .1001, .1592, also show that the Jagiellonian game is closer to the Nice game than to the Lisbon game.

Table 3
The voting power of Germany in three games under the naive and classical approach

Power measure \ Game	Nice (G_1)	Jagiellonian	Lisbon
Integer weight	29	33	58
Relative weight	.0841	.0957	.1673
Banzhaf index	.0778	.0955	.1164

6.2. The naive way of measuring blocking power

The naive way of theorizing which equates voting power with relative weight deserves its name "naive" – I must say as a *mathematician*. Yet I am a *social scientist*

too. As such I would not reject this approach altogether, even if it was dismissed by Felsenthal and Machover (1998, p. 156) as “widespread fallacy” to which “even experts on voting power are not immune.” The way in which the players *themselves* calculate how strong they are may affect the results of the game. It may well be that the members of a voting body who, for their weights, are assigned high *political status* can easier find partners for winning coalitions and thus enjoy a greater *political power*, that is, they have a greater *real* influence on collective decisions.

Politicians and their advisors – experts in constitutional law or non-mathematical political science– hardly ever go beyond naive theorizing. Since they have always been preoccupied with maximizing blocking power, they must have invented their own measure computed from the weights and quota. The naive coefficient of blocking power, known as the *share of blocking minority*, is defined as the ratio of a voter *i*’s weight p_i to the *blocking threshold*. I learned about its common use by EU politicians from newspaper reports and Moberg’s paper (2007).

To explain what is blocking threshold, notice that in a voting game with weights p_1, \dots, p_n and quota q , the type of any coalition C can be easily determined by finding out which of three successive intervals contains the *total weight* $p(C)$ of C , or the sum of p_i over all members of C . The intervals which correspond to losing, blocking, and winning coalitions have the form:

$$[0, p(N)-q], \quad (p(N)-q, q), \quad [q, p(N)].$$

The lower bound of the middle interval, or $p(N)-q$ is usually referred to as the *blocking threshold*. When the weights and quota are integers, it is more convenient to define this quantity by means of the formula $r=p(N)-q+1$, which implies that any coalition C is blocking if and only if $r \leq p(C) < q$.

For example, for the weighted voting game used in EU-15, we have $p(N)=87$, $q=62$, so that $r=87-62+1=26$. Hence, for 4 strongest members of EU-15 (each of them received 10 nominal votes in the Council), the share of blocking minority equals $\frac{10}{26}=38.5\%$. The dissatisfaction of the Big Four with the outcome of negotiations in Nice might have had to do with the fact that their share of blocking minority, computed for the game with political weights (G_1) dropped to the value $\frac{29}{91}=31.9\%$. The game with population weights (G_3) was added to compensate for this loss, but only Germany benefited from making voting rules more complex ($\frac{167}{381}=43.8\%$; for France we have: $\frac{128}{381}=33.6\%$). For Spain, the shares of blocking minority, computed for the game used by the “old Union” and for the political component of the game designed for the enlarged Union marginally differ ($\frac{8}{26}=30.8\%$, $\frac{27}{91}=29.7\%$), both being much greater than the value ($\frac{89}{381}=23.4$) for the population component of the Nice game.

How to estimate Spain's total blocking power in the game that was defined as the intersection of three weighted voting games? Did that country get more power than it had in the Fifteen? The naive conception of winning and blocking power is unable to answer such questions. In addition, it yields odd results even for some voting games to which it can be applied. Compare two historical weighted voting games, the one used by the original Six and the other designed for the Nine which replaced the Six. In the first game, the value of the naive coefficient of blocking power for Luxembourg was equal to $1/6 = 16.7\%$ despite the fact that this player did not belong to any minimal blocking coalition. By contrast, in the second game, Luxembourg is a member of 4 minimal blocking coalitions of size 4, but its share of blocking minority equals $2/18 = 11.1\%$.

6.3. The classical conception of (winning) power vs. the one based on the idea of blocking

The practice of constructing voting systems by the politicians and their anonymous experts has always appeared to academic specialists devoid of any theoretical foundations. Felsenthal and Machover (2009, p. 321) admit that "politicians are keenly interested in negative or blocking power – the ability to help block an act that they oppose," but "this does not mean that they have more than a vague notion as to how to quantify this power." Such an opinion seems to me a bit exaggerated. Amateurish study of blocking power has in fact gone beyond the limits of the naive approach I've just described. But it is true that insights and preconceptions behind tinkering with the blocking threshold and counting small size minimal blocking coalitions have so far remained without an adequate formalization. The aim of Chapters 3–5 of this paper was to make a significant step in this direction.

Analysts attached to the classical approach find it astonishing that the negotiators at EU summits are ready to argue till dawn about raising the quota by few points or appending an odd-looking clause to the treaty. Does it make any sense – they ask – to quarrel, once such minor modifications of the rules of the game negligibly affect the values of the "scientific" measures of voting power. Indeed, let us compare the double majority game $H_1 \cap H_2$ (H_1 is the population game with quota 650 and H_2 is the 1 voter – 1 vote game with quota 15) with the Lisbon game which has the form $H = (H_1 \cap H_2) \cup H_3$ (H_3 is the 1 voter–1 vote game with quota 24). If you compute the Banzhaf index for these two games of which the second differs from the first only with the *ban on blocking in threes* (this rule is formalized by defining H as the union of $H_1 \cap H_2$ and H_3), you will find out that the difference in power will not exceed .0001 for any player. But, as I'm going to show later in this paper, the condition that a *blocking minority must include at least four Council members*, has a dramatic effect on the blocking structure.

I have long wondered why the constructors of EU voting games have been so little interested in the classical approach to voting power. Now I know that its too weak reception does not result from its mathematical sophistication. Does the concept of a critical player appear to laymen more obscure than that of a blocking minority? Certainly, not. I agree with Moberg (2007) that classical indices proved of little practical value because their calculation is based on the assumption that, in estimating a player's *winning* power, *millions* of theoretically possible and *equally probable* winning coalitions containing him as a critical member must be taken into account. Actually, what the players want to maximize is not winning but *blocking* power, and what really matters for a player is to find alternative partners to form *small* minimal blocking coalitions of which the number is counted in hundreds rather than millions. Unlike numerous "anonymous" winning coalitions, many small "blocking minorities" can be quite concretely identified by the players for the sake of their political rather than mathematical calculations. What the politicians would like to know is how to find allies for blocking the initiatives of their rivals or whose support to seek to prevent blocking their own proposals.

The classical formalization of winning power (wrongly equated with voting power *tout court*) and the study of blocking power are, in fact, two branches of one *axiomatic mathematical theory*. The "technologies" they generate are complementary to each other, yet in some cases they may prompt different practical solutions, first of all, as to the quota selection.

The next chapter offers a brief history of constructing voting games for the Council of Minister of the EU. It's going to be a *verstehende Geschichte*, as Max Weber would say. I will try to decipher the intentions of the constructors by analyzing the "architecture" of the "cathedrals" they built. My claim is that their aims become understandable when their actions are seen in light of the approach developed in the first part (chapters 2-5) of this paper. The descriptions of voting systems and population data used in the analyses given in Chapter 7 come from Chapter 5 of Felsenthal and Machover's book (1998).

7. A HISTORY OF VOTING SYSTEMS CONSTRUCTED

FOR THE EU COUNCIL OF MINISTERS

7.1. From the Six to the Twelve

7.1.1. How the story began some fifty years ago. The germ of today's European Union consisted of 3 large states, 2 much smaller states, and 1 tiny state whose citizens then formed some 0.2% of the total population of the Six. Luxembourg got 1

nominal vote in the Council of Ministers, the Netherlands and Belgium – 2 votes each, 3 largest states – 4 votes each. The population of France (then the smallest country among the top Three) was almost exactly 4 times greater than that of the Netherlands, but the proportion of weights assigned to the two states was not 4 but 2, or the *square root* of 4. The largest proportion of populations within the Three was equal to 1.1. Although this could have been the sufficient reason to give the same weight to the top players, the decision of the father founders of the European Community was *political par excellence*. The *parity principle* was also applied to the second group: Belgium and the Netherlands received the same number of nominal votes.

With such an allocation of weights, 11 is the lowest winning threshold which both allows the Big Three to outvote the Benelux¹⁰ and prevents any two out of the Big Three from outvoting the third one with the help of one of three smaller players. However, with such a quota the three weaker actors would be structurally interchangeable (indeed, if the Netherlands or Belgium is replaced with Luxembourg in any winning coalition, a winning coalition is obtained again) and, consequently, they would have the same value of any structural parameter. The constructors of the first voting game for the Council of Ministers could easily discover this fact by examining the set of all coalitions, which was not a too difficult task, since their number equals only $2^6=64$.

The quota was finally set at 12 votes. Belgium and the Netherlands gained power advantage over Luxembourg but the smallest member of the Six was deprived of any winning or blocking power. Nevertheless, the voting game so obtained had the property which could have been considered desirable by the constructors, namely, the construction resulted in dividing the set of 6 actors into three subsets {1,2,3}, {4,5}, and {6}, such that the actors in each of them are structurally interchangeable, while those from different subsets are not so.

In general, the construction problem can be stated abstractly as follows: for a given partition of the set N of actors, construct a game such that pairwise disjoint subsets which form this partition coincide with the equivalence classes generated by the relation of structural interchangeability. The solution of the problem can be sought by inspection of all non-isomorphic voting games with n players. For $n=2$ or $n=3$ this can easily be done¹¹, but for larger n one has to resort to a computer program that will generate non-isomorphic games, determine for each its automorphisms

¹⁰ Equivalently, this condition means that Benelux was denied the right to block initiatives agreed-on by the three bigger members of the Six.

¹¹ For $n=2$ there exist only 2 structurally distinct voting games: the *consensus* game and the *dictatorial* game in which one of two players forms a unique minimal winning coalition. For $n=3$, there are 5 nonisomorphic games, two named above, and three other: the *duumvirate* game (two players form the only minimal winning coalition), the *hegemony* game (there are two minimal winning pairs having one voter in common; the latter has the right of veto, but needs cooperation of one of two other players to form a winning coalition), and the *majority* game (all three pairs are minimal winning coalitions).

(they form a *group* in the algebraic meaning of the term) and equivalence classes of structural interchangeability. If the problem is solved positively, that is, there exist voting games that meet the required condition, one can take such a family of games as a pool from which further selection is to be made: by imposing further (logically consistent) requirements, such as those probably considered by the constructors of the game for the Six (in particular, they decided to endow any two members of $\{1,2,3\}$ with blocking power). They started from assigning weights to try next various quotas until they decided to stay with $q=12$, having noticed that such a quota generated the required *structural partition* of the set of players. The (probably) unintended effect for Luxembourg (this state could only persuade other states to vote according to its preferences; how it voted had no influence on the outcome of any voting) could have been avoided, but this would require a change in the assignment of weights.

7.1.2. The first two enlargements. The position of Luxembourg changed to better when Great Britain, Ireland and Denmark joined the Six. In a new allocation of nominal votes, which was introduced for the Nine, old proportions of weights were preserved between two upper groups. To mark the difference between the second group and the third group containing two smaller countries, Denmark and Ireland received 3 votes. Luxembourg, with its outlying population size, formed now the fourth, one-element group. The four largest states were given 10 votes each, which value was both large enough to allow for more steps in the ladder of weights and convenient for calculating the ratios of weights.

It is natural to assume that any winning coalition in the game to be constructed should be at the same time a winning coalition in the simple majority 1 state–1 vote game, that is, for $n=9$ it should consist of at least 5 states. The minimum quota under which this condition is met equals 41. It is the value which was actually used by the constructors of the game for the Nine.

Our advice for the reader at this point is such: Return to Section 6.2 before you analyze the content of Table 4, in which section you will find the explanations of the symbols used in the headings of the columns. Here we shall only recall that $p(N)$ is the sum of p_i over all $i \in N$ and $r = p(N) - q + 1$, or the *blocking threshold* in a weighted voting game with *integer weights*, is the number, smaller than q (the quota), such that C is a blocking coalition if and only if $r \leq p(C) < q$, where $p(C)$ is the sum of p_i over all $i \in C$. See also Section 3.2 to recall that $b_{m,k}(i)$ is the number of minimal blocking coalitions of size k containing actor i .

Table 4
Blocking structure in the games for the Nine, Ten, and Twelve

n	$\rho_1 \dots \rho_n$	$\rho(N)$	q	r	$b_{m,k}(i)$		
					$k=2$	$k=3$	$k=4$
9	$\rho_1 = \rho_2 = \rho_3 = \rho_4 = 10$ $\rho_5 = \rho_6 = 5$ $\rho_7 = \rho_8 = 3$ $\rho_9 = 2$	58	41	18	3	5	1
					0	12	0
					0	8	4
					0	0	4
10	$\rho_1 = \rho_2 = \rho_3 = \rho_4 = 10$ $\rho_5 = \rho_6 = \rho_7 = 5$ $\rho_8 = \rho_9 = 3$ $\rho_{10} = 2$	63	45	19	3	3	9
					0	8	12
					0	0	24
					0	0	24
12	$\rho_1 = \rho_2 = \rho_3 = \rho_4 = 10$ $\rho_5 = 8$ $\rho_6 = \rho_7 = \rho_8 = \rho_9 = 5$ $\rho_{10} = \rho_{11} = 3$ $\rho_{12} = 2$	76	54	23		28	19
						22	16
						10	39
						6	32
					0	8	

When Greece became 10th member of the Community, it joined the second group and received the same number of votes as Belgium and the Netherlands. The quota 45 was used instead of 41, or the minimum threshold guaranteeing that any winning coalition must have $\frac{1}{2}n$ members. Why? With $q=41$, the blocking threshold equals $63-41+1=23$, which implies that any minimal blocking coalition must have at least 3 members. In the Six and the Nine, the minimum size of a blocking coalition was 2. In addition, in the Nine, the right to block in pairs was reserved for 4 top players. Under the weights used in the Nine and the Ten, the lowest blocking threshold with such a consequence equals 16 (if r were smaller, then any pair of players with weights 10 and 5 would be a blocking coalition). On the other hand, the highest blocking threshold which still enables blocking in pairs equals 20. Thus, one had to choose as quota for the Ten a number from the range from $q=44$ ($r=20$) to $q=48$ ($r=16$). Since quotas 44 and 45 generate the same set of winning coalitions, either value could be used to define the game. Since 44 cannot be attained, 45 was used.

7.1.3. Spain joins the Union, or first problems with extending the game. When Spain and Portugal joined the Union (1986), Portugal was added to the group of 5-vote states. Spain's population (38.6 million) was then much closer to France's (54.1) than to the Netherlands' (14.6), so the club of the most powerful states could admit Spain for 5th member. Otherwise one had to add a step in the ladder between the four big and four middle size states. Without changing the collection of weights, this could have only been done by assigning to Spain 6, 7, 8, or 9 votes. If two extreme numbers

are discarded, one has to choose between 7 and 8. The choice of the greater number can of course be interpreted as Spain's political victory, but this country's ability to win negotiation games is not the only plausible explanation of why 8 was chosen.

Let us take the population of the largest country in each of 5 groups which make up the set of 12 states for the basis for determining the common weight for the group. Let 10 be the maximum weight, or the number of nominal votes granted to members of the top group. Table 5 shows two ways of assigning integer weights to the remaining groups. The first method is based on the postulate that the proportions of weights should be as close as possible to the respective population ratios. For example, since the population of Spain (38.6 million) was then 63% of the population of West Germany (Bundesrepublik had some 61.0 million citizen before absorbing DDR), Spain should obtain 10 times .63, or 6.3 rounded to 6 nominal votes (if the group mean were taken to represent the group, the ratio would equal .67, which translates to 7 votes). Under the second method, which uses square roots of the populations, the ratio equals $6.21/7.81 = .795$, which yields 8 votes for Spain.

As shown in Table 5, the weights which were actually assigned to 12 states may have been calculated by means of the square root method, the case of Luxembourg being the only exception.

Theoretical reasons for the use of square root weights will be discussed in Chapter 8 of this paper. Now let us try to guess how the top players might approach the problem of determining the quota for the Twelve. Once they agreed to give to Spain 8 votes, it seemed unlikely that they would make further concessions. Therefore, the Big Four should have demanded that r be equal to 19 or 20 ($q=58$ or 57) in order both to exclude Spain from blocking in pairs and to guarantee this privilege to themselves. Quite unexpectedly, as if community spirit overcame greed for power, the quota was set at $(4 \cdot 10 + 8 + 5) + 1 = 54$, or the minimum number such that any winning coalition must consist of 7 states. As a consequence, since $r=23$ for $q=54$, the minimum size of a "blocking minority" was raised to 3.

Table 5
Two methods of determining integer weights from the population data

Group	Country	Pop.	Ratio	Pop. wght	Sqrt pop.	Ratio	Sqrt wght
1	Germany	61.0	1.00	10	7.81	1.00	10
2	Spain	38.6	.63	6	6.21	.80	8
3	Netherlands	14.6	.24	2	3.82	.49	5
4	Denmark	5.1	.08	1	2.26	.29	3
5	Luxembourg	.4	.01	0	.63	.08	1

The blocking structures in the games designed for the Nine, Ten, and Twelve are displayed in Table 4. All of them are multilevel, but none of them is regular. The lack of regularity can be explained in two alternative ways. Political decision-makers may have been interested only in the lowest level of the blocking structure. In estimating blocking power, they simply didn't take into account larger coalitions. But it may well be that irregularity was consciously approved as a way to ensure balance of blocking power between stronger and weaker players. The latter may have been granted more opportunities to block in larger coalitions to compensate for being denied access to smallest size blocking coalitions.

Voting games can be classified into four types with respect to the shape of the premier league. The premier league can be *exclusive* or *inclusive*, and *hierarchical* or *egalitarian*. For the EU games, exclusiveness can be operationally defined by the condition that the premier league contains at most 1/3 of all players. All configurations, except the inclusive-egalitarian type, occur in the history of EU games. The historical importance of the game designed for the Twelve consists in the transition from the exclusive-egalitarian type to the inclusive-hierarchical type. In the Twelve, the premier league consists of 11 players (only Luxembourg cannot participate in a blocking three), but the distribution of the number of blocking threes is uneven.

7.2. The case of the Fifteen, or it is possible to construct a voting game with regular blocking structure

The blocking structures in 4 voting games which were probably considered in designing a voting system for the Fifteen are displayed in Table 6. The sum of the weights of 7 largest countries now equals 58. Therefore, to avoid constructing a "double majority voting system" (in our terminology, intersection of two weighted voting games), one should try quotas from 59 upwards. The quota actually used was 62. Why? And why the smallest relevant quota was not used?

Two columns under G_1 show the numbers of minimal blocking coalitions of size 3 and 4 containing each of 15 states. Only the Big Four is granted the right to block in threes. Games G_2 , G_3 , and G_4 extend this right to Spain. Why games G_2 and G_3 were rejected and G_4 was found acceptable? All three games have the same set of blocking threes. The differences appear on the second level of the blocking structure, that of blocking fours. Notice that G_2 and G_3 unlike G_4 do not meet the condition of regularity to the disadvantage of Spain. What a mathematical-political scientist cannot guess without consulting political actors involved is only whether a smart expert working for Spain outwitted the Big Four, or 4 strongest players agreed to admit Spain to the premier league as well as to give Spain the right place on the second level of the blocking structure.

Table 6
Blocking structures in 4 voting games for EU-15

Member state	Weight	$G_1: q_1=59$		$G_2: q_2=60$			$G_3: q_3=61$		$G_4: q_4=62$	
		$k=3$	$k=4$	$k=3$	$k=4$	$k=5$	$k=3$	$k=4$	$k=3$	$k=4$
1. Germany	10	3	72	6	87	145	6	125	6	153
2. France	10	3	72	6	87	145	6	125	6	153
3. UK	10	3	72	6	87	145	6	125	6	153
4. Italy	10	3	72	6	87	145	6	125	6	153
5. Spain	8	0	60	6	24	281	6	56	6	108
6. Netherlands	5	0	36	0	60	137	0	74	0	86
7. Greece	5	0	36	0	60	137	0	74	0	86
8. Portugal	5	0	36	0	60	137	0	74	0	86
9. Belgium	5	0	36	0	60	137	0	74	0	86
10. Sweden	4	0	30	0	30	184	0	64	0	74
11. Austria	4	0	30	0	30	184	0	64	0	74
12. Denmark	3	0	6	0	24	170	0	36	0	64
13. Finland	3	0	6	0	24	170	0	36	0	64
14. Ireland	3	0	6	0	24	170	0	36	0	64
15. Luxembourg	2	0	6	0	0	138	0	24	0	36

The history of designing voting games for the EU Council abounds in dramatic turns, but it is by no means so absurd as it has always appeared to the mainstream analysts who have been able to notice no more than that the *relative* quota varied around 71% and that the constructors simply reproduced the pattern that had been used for the first time for the initial Six (where $q/p(N)=^{12}/_{17}=70.6\%$).

In the language of our non-classical approach, the transition from the game for the EU-12 to the game for EU-15 can be characterized as replacing the premier league of the inclusive-hierarchical form with the one falling under the type which was called here exclusive-egalitarian.

Smaller states, which lost the privilege of blocking in threes, were dissatisfied with such a change, however. They gained support from the UK, which resulted in what became known as the *Ioannina compromise*, or the provision that the old blocking threshold (23 votes) will remain in use, but only for meta-deciding on whether a proper decision arrived at by applying the new voting system shall take effect immediately or shall be suspended for some “reasonable” time to allow for more discussion and possible revision of the original act.

7.3. From the Nice treaty to the Lisbon treaty

7.3.1. Nice weights. A historical-theoretical analysis of the Nice voting system must begin from an attempt to explain why those who designed the voting system for EU-27 found it necessary to discontinue the practice of assigning weights to new members by appending them to already existing groups (in the previous enlargement,

Finland joined Denmark and Ireland) or occasionally creating a new group (Sweden and Austria were given 4 votes each, the only integer between 3 and 5). Such a procedure could have been applied to 12 states, but then 3 very small countries, Latvia, Slovenia, and Estonia, for the lack of intermediate integer between 2 and 3 had to join the last group (with Cyprus, Luxembourg and Malta) or the second one from the bottom, made up of countries with population ranging from 3 to 5 million. However, a more serious problem was the necessity to place too many countries with too widely varying population numbers in the group whose original members were only Belgium and the Netherlands. The discussion of this problem at the negotiation table might have inspired Germany (its population after the unification far exceeded the population of each of 3 other largest EU countries) to demand a change in the allocation of weights within the strongest group as well.

The system of weights based on 10 as the maximum value was invented for 9 states, so it was quite natural to take 30 as the baseline weight for a voting system for the Council with 27 members. If the parity principle were retained and old square root proportions of weights preserved (the population of Germany being counted as before the unification), then the allocation of weights on the top would be: {Germany, France, UK, Italy} – 30 votes, {Spain, Poland} – 24 votes, and {Romania, Netherlands} – 15 votes. Now I must warn the reader that they may find my analysis too speculative. Indeed, it departs from the historical account presented by Rafał Trzaskowski in his monograph (2005: 198–214). Trzaskowski took great pains to reconstruct from available documents the story on how the Nice voting system came into being as the outcome of a process of long, tedious, somewhat chaotic negotiations. Reporting on all what happened in Nice and might have effect on the final result, he refrained from interpreting some facts he had found in the sources as links in certain meaningful chains of actions or events nor did he try to offer explanations that would invoke definite goals or strategies attributed to the actors. His *historical* account, however, is not entirely free from theorizing. The author did not overlook the importance of blocking for those who designed voting systems. My approach, which consists in applying *a theory* to history to make it understandable, gives priority to reconstructing the „choice space” the constructors of EU voting systems have considered at each stage of their work. Tentative answers are offered too – wherever possible – to the question of why these and not other solutions were accepted. The constructions themselves – rather than negotiation records or interviews with the politicians – have been for me the basic material for *theoretical* analysis.

Thus, my conjecture as to why strange numbers 29 and 27 appeared on the top of the list of Nice weights points to taking away 1 vote from the numbers originally assigned to 4 stronger states (Germany, France, UK and Italy) and 1 vote from the numbers assigned to 2 weaker players (The Netherlands, Romania), which gives

6 extra votes to be divided evenly between Spain and Poland. The fact is that the constructors of the game for EU-27 approved of a serious distortion of the square root proportions. It is not clear, however, whether they were aware of what they did and whether they realized that their decision meant a break with a practice that “worked” in the past (see Table 4 in Section 7.1.3).

7.3.2. Three variants of the Nice game. The next step in constructing the game for EU-27 was the choice of a quota. As previously, largest weights (now of 13 players) were added up to determine $q=257+1=258$. The respective $r=345-258+1=88$ turned out to be the smallest blocking threshold under which 4 became the minimum size of a blocking coalition.

The first variant of the Nice game defined in such a way has one-level blocking structure. Players 1–27 participate in 411 blocking fours with the following frequencies (given in brackets): 1–4 (214), 5–6 (196), 7–21 (20), 22–27 (16). The premier league containing all players consists of two groups of unequal size. While within-group differences are very small, the Big Six has gained enormous power advantage over the remaining 21 states. Who was more disappointed at the “two-class model” of political structure planned for EU-27? The Big Four, which had to acknowledge growing aspirations of the Semi-big Two, or the Twenty One fearing the domination of the Big Six? As it were, in the next phase of the negotiations, the blocking threshold was changed. We would not have learnt that $q=258$ had been considered, hadn’t the negotiators left this number in the text of the treaty along with the new blocking threshold. Interestingly, contrary to the custom of concealing the real “methodology” behind legal formulations, the number 91 appeared explicitly in the treaty as the minimum size of a “blocking minority.” Since for $r=91$ we have $q=255$, there appeared an inconsistency which was noticed by classical theorists who were first to analyze the Nice game (Felsenthal & Machover, 2001).

It is not clear why the old blocking threshold was replaced by 91 rather than another number greater than 88. 91 is perhaps a trace of the original hypothetical assignment of weights. If four largest states are assigned 30 votes each, then 91 is the smallest threshold under which blocking in threes becomes impossible.

Since the winning threshold of 255 votes can be attained by few coalitions having less than 14 members, the negotiators found a remedy in adding the requirement that a “qualified majority” must comprise at least 50% of the members of the Council. That’s how was born the second variant of the Nice game – the first “double majority” voting system in the history of the EU. The premier league now included all players except Malta. The distribution of 315 blocking fours (1–4 (170), 5–6 (140), 7–21 (20),

17–21 (16), 22–26 (4)) differs from the one obtained for the first variant only with greater inequality within the Big Six.

That is not the end of the story. The third, final variant, or a “triple majority game”, was produced by putting into the already complicated system another component, the qualified majority game, now with population weights and relative quota 62%. The population component was added to give to the Big Four the privilege of blocking in threes. Spain, owing to its enormous population growth, joined the premier league soon (2003) after the Nice treaty was signed (2001). The premier league, which since then until today consists of 5 players, is the first example in the history of the EU of combining high degree of exclusiveness with within-group hierarchy.

7.3.3. The Convention game and the EU Constitution (Lisbon) game. Soon after the final version of Nice treaty was signed (February 2001) a somewhat mysterious body known as the Convention designed a new voting system to be included in a new treaty that was to replace the Nice treaty. The game with “political” weights was discarded, which was justified by the need to simplify the hybrid system rightly perceived to be too complicated. In addition, in the double majority game which was left, the quota in the population component was lowered from 62% to 60%. The first effect of the latter decision was the exclusion of Spain from the premier league, the second – making Germany’s advantage over France, UK and Italy more permanent in the situation when Three’s share of the total population of EU-27 was approaching 37%, a value too close to the Nice threshold of 38%.

In Poland, the change of the population quota went unnoticed. Poland’s angry reply (*Niza o muerte* was said by a member of Polish parliament in order to encourage the government to defend the Nice treaty) to the Convention’s proposal was a result of the disappointment that the idea of the Big Six (easy to read from the numbers 29, 27, 14 alone) was put to death soon after its birth. In addition, the draft of the Constitution treaty became known soon after the accession referendum which had been won by the pro-European forces. Poland’s relative *political* weight $27/345=7.8\%$ roughly coincided with the share of the total EU-27 population. Why, therefore, not to get rid of arbitrary political weights and keep the population component only, once Poland has in fact an equally strong position in both components of the Nice game? That’s how the naive approach to voting power could have been employed to persuade Polish political elites to approve of the voting system proposed by the Convention. However, when Polish classical theorists calculated the Banzhaf index for the Nice and the Convention game and showed the bar charts to the public, over 100 eminent scholars and intellectuals signed (November 17, 2003) an appeal to stand by the Nice Treaty.

Most experts expected that the quota in the population game would be set below the level proposed by the Convention. Actually, June 18, 2004, the EU summit did the contrary by setting the quota at 65%. However, the distribution of the number of blocking threes across 6 states which form the premier league turned out extremely hierarchical. In addition, Germany could enjoy membership in 9 out of 10 blocking threes, by far surpassing other largest countries in this respect. As a consequence, the Inter-Governmental Conference ruled in the last minute that the following clause will be put into the Constitution treaty: *A blocking minority must include at least four Council members, failing which the qualified majority shall be deemed attained.*

Table 7
Blocking structures in the Double Majority Game ($q_1=14$, $q_2=650$) and the Lisbon game (DMG with the ban on blocking in threes)

EU-27 member states	Double Majority Game			Lisbon Game	
	$b_{m,3}(i)$	$b_{m,4}(i)$	$b_{m,5}(i)$	$b_{m,4}(i)$	$\gamma(i)$
1. Germany	9	30	363	229	79.8
2. France	5	36	353	149	51.9
3. UK	5	29	287	142	49.5
4. Italy	5	27	238	140	48.8
5. Spain	3	37	141	107	37.3
6. Poland	3	17	226	87	30.3
7. Romania	0	28	125	38	13.2
8. Netherlands	0	11	172	21	7.3
9. Greece	0	8	101	18	6.3
10. Portugal	0	7	101	17	5.9
11. Belgium	0	7	101	17	5.9
12. Czech R.	0	7	101	17	5.9
13. Hungary	0	5	119	15	5.2
14. Sweden	0	5	100	15	5.2
15. Austria	0	3	114	13	4.5
16. Bulgaria	0	3	104	13	4.5
17. Denmark	0	0	84	10	3.5
18. Slovakia	0	0	84	10	3.5
19. Finland	0	0	84	10	3.5
20. Ireland	0	0	58	10	3.5
21. Lithuania	0	0	52	10	3.5
22. Latvia	0	0	36	10	3.5
23. Slovenia	0	0	31	10	3.5
24. Estonia	0	0	22	10	3.5
25. Cyprus	0	0	16	10	3.5
26. Luxembourg	0	0	11	10	3.5
27. Malta	0	0	11	10	3.5
	10	65	647	287	

Table 7 shows that the Double Majority Game has a highly irregular blocking structure. Appending the ban on blocking in threes to DMG results in guaranteeing to every player an opportunity to block in fours. Thus, the blocking structure has

one level and the sequence $b_{m,4}(i)$ is consistent with population weights. However, the distribution of the number of blocking fours is very uneven. Germany takes the dominating position, which is also seen if you calculate the Banzhaf index.

If you use the measure γ , you will notice that the largest difference in blocking power between two states occupying neighbor positions in the ordering occurs between Germany and France. Paradoxically, it was Poland not France that seemed more dissatisfied with replacing the Nice game by the Lisbon game.

8. BLOCKING POWER IN SQUARE ROOT GAMES

8.1. The rationale for the use of square root weights

8.1.1. The two-tier voting game. Under the naive approach to voting power, the replacement of population weights with their square roots can be justified solely as a means aimed at flattening the power distribution. However, inter-actor differences with respect to relative weight can be reduced by applying various mathematical functions to original population weights. The rationale for the choice of the square root function was offered long time ago by Lionel Penrose (1946) who not only laid the foundations of the *classical* understanding of voting power, but proposed a *two-tier voting game* as a formal model of *indirect* voting. His model is briefly described below after Felsenthal and Machover (1998).

Assume that the set N of voters (let us call them *citizens*) consists of m pairwise disjoint sets (referred to as *constituencies*) N_1, \dots, N_m . When a proposal is put to the vote, citizens decide first independently in each constituency N_i by means of a voting game with some \mathcal{W}_i as a set of winning coalitions. If the set C_i of members of N_i who voted for the proposal is a “locally” winning coalition ($C_i \in \mathcal{W}_i$), then the *delegate* d_i of i th constituency is bound to vote “yes” when the *council* of delegates $M = \{d_1, \dots, d_m\}$ is to decide on the issue with the use of its own voting rules formalized as (M, \mathcal{V}) . The set \mathcal{W} of winning coalitions in the *two-tier voting game* over N consists, by definition, of all subsets C of N such that the delegates who represent the constituencies N_i in which $C_i = C \cap N_i$ is a winning coalition form a winning coalition in the game (M, \mathcal{V}) . Formally, $C \in \mathcal{W}$ if and only if $A(C) \in \mathcal{V}$, where $A(C) = \{d_i; C_i \in \mathcal{W}_i\}$.

8.1.2. Square root laws. Penrose studied two-tier voting games which have 1 citizen–1 vote simple majority games on the bottom and a weighted voting game on the top. In a 1 voter – 1 vote simple majority game, a subset C_i of N_i is a winning coalition if and only if $|C| > n_i - |C|$ where $n_i = |N_i|$. Every game of the kind is *symmetric* (any two

players are structurally interchangeable), and so is the 1 voter–1 vote simple majority game with the set of players $N=N_1\cup\dots\cup N_m$. The latter game is a model of *direct* democratic decision mechanism in assembly N . Symmetry implies, in particular, that all players are granted *equal* theoretical influence on collective decisions. Penrose showed that it is possible to construct a system of *indirect* voting which approximately meets the same condition of equality. His theorem (Th. 3.4.3 in Felsenthal & Machover, 1998, p. 66) states that if all n_i are sufficiently large, then the two-tier voting game of the form described above has an approximately even power distribution if and only if the power of each delegate in the game (M, ν) is roughly proportional to the square root of the size of his constituency, or equivalently, for any two delegates d_i and d_j , the ratio of their voting powers negligibly differs from $\sqrt{n_i}/\sqrt{n_j}$. The voting power of a citizen or delegate was quantified by Penrose by means of the number of winning coalitions containing a given player as a critical member.

Felsenthal and Machover (1998, p. 68) noticed that Penrose’s theorem, usually referred to as the “(first) square-root law” is often misstated by saying that all citizens have equal voting power in a two-tier game if the *weights* in the game (M, ν) are proportional to the square roots of n_i . In the correct statement, proportionality is required of the delegates’ voting powers not weights. However, Słomczyński and Życzkowski (2006, 2007) showed that voting power can be made approximately proportional to the square root weight by setting the quota at a special value they called “optimal.” To justify the use of $\sqrt{n_i}$ instead of n_i , one can also invoke the theorem known as the “second square-root law” (see Felsenthal & Machover, 1998, p. 72–78).

It seems unlikely that EU leaders would ever accept a *unique* quota derived from a formula which has solely scientific reasons behind it. The politicians will never renounce the control over the quantity which under fixed weights determines the set of blocking coalitions. As regards square root weights, there is more hope that their mathematical underpinning will not be disregarded.

Prior to considering practical applications of the square root laws, one must understand the difference between indirect representation of EU citizens in the Council of Ministers of EU by their governments and their indirect representation in the European Parliament by the sets of deputies elected in particular countries. Since the members of any parliamentary group are not bound to vote unanimously, national factions, unlike governments, must not be considered as players in a voting game. Therefore, there is no reason to assume in advance, as do naive EU democrats, that the number of indivisible votes in the Council and the number of individual votes in the EP should be related to a state’s population by the same linear function.

The supporters of the Constitution voting system who pointed to its “democratic” character should think over the following fictitious, yet by no means unrealistic story.

Suppose that all citizens of the EU are asked the question: “Do you agree to give more power in the Council to the governments representing large states, that is, those having each more than 10% of the EU total population?” Imagine now that a fictitious social movement “Europeans for Democratic Europe” launched a campaign against giving more power to the 4 largest states. As a consequence, the frequency of “yes” in each of them remained still high, but it dropped to 70%, while in each of 23 smaller states it attained the level of at most 20% following an opposite campaign based on the slogan “Give more power to the Four so that they could help you more.” Since the citizens of the Big Four make up today some 54% of the total EU population, the proposal would fail in a referendum, for having gained support of no more than $0.54 \cdot 0.70 + 0.46 \cdot 0.20 = 0.47$, or 47% of all EU citizens. What would be the outcome of an indirect voting? If all governments were to respect the results of the poll in their states, and the states’ shares of the total EU population were taken as weights in the voting game to be played by the Council, then the proposal would be passed, even if it is backed by a minority of EU citizens. If the general simple majority game with square root weights is used on the upper tier of a two-tier voting system, then the outcomes of direct and indirect voting are more likely to be identical. In our concrete case, the proposal, which was backed by 4 largest states only, will fail since the sum of square root weights assigned to them is some 34% of the total. One can try, of course, other ways to bring back the power to the EU citizens (raising the winning threshold in the Council and/or adding a 1 delegate–1 vote game with a proper quota), but why not to use square root weights which provide the simplest solution to the problem.

8.1.3. The EU member states still have retained much of their sovereignty, first of all, in the domain of international politics. However, with narrowing down the range of issues decided by the *consensus game*, the EU will evolve toward a *federation* of states. With blocking getting more and more difficult (because of increasing minimum size of minimal blocking coalitions in voting games that are used to decide where unanimity is no longer required), the EU leaders should be more and more concerned about measuring their winning power and maximizing efficiency of the system of collective decision-making. They should finally appreciate the virtues of the square root function which is believed by the theoreticians to be the most natural way of assigning weights to the delegates of autonomous political communities which have been united into a federation while *varying in size*.

Interestingly, the use of square root weights was not supported by those EU members which had appeared to favor advanced integration of the union of states. Paradoxically, it was Poland – a country where the idea of transforming the EU into a federal superstate still faces strong resistance – that engaged in a “battle for the

square root” with France and Germany. It is no less paradoxical that the square root mapping of the population structure into weights has been denounced by many Polish mainstream pro-EU political commentators as a purely theoretical or even magic idea, whereas, as I have shown in Chapter 7, some square root proportions between weights were already in use in the pre-Nice voting systems, albeit it is doubtful if their constructors have been inspired by Penrose’s theory of two-tier games.¹² The Nice treaty brought a distortion of the proportions between nominal votes within the Big Six, which error was “miscorrected” by adding the game with population weights in order to give, in a roundabout way, more power to Germany. The Lisbon treaty went even further by making the population distribution the basis of a new voting system, which implied rejecting the parity principle. The “demographic principle” accepted at the EU summit in 2007 has still been defended, yet the main argument which has since then been raised against returning to the debate on voting rules for the Council of Ministers became: “Let’s stay with the system we arrived at after a so long row in order not to open the Pandora’s box once again.” The EU leaders, who had got scared of the unknown, as they tended to perceive the “square root system,” may have been unaware of the hegemonic nature of the system they finally chose. Anyway, they decided to stay until 2014 with the familiar Nice voting system to which even its opponents had already got used.

8.2. Two politically interesting square root games

Why did the Big Four approve of the Lisbon “compromise” that was probably sought by Poland from the outset? We show in this section that if the strongest EU states accepted square root weights, they could gain more blocking power than they had in the Nice game and no less than in the Lisbon game. The prejudice against square root weights probably stemmed from relying solely on the naive understanding of voting power. Indeed, if relative square root weights were used instead of relative population weights, Spain and Poland’s position on the power scale would come closer to Italy’s. However, if “power” means “blocking power” measured by means of the number of smallest size minimal blocking coalitions containing a given player, then how great is the power advantage of the Four over the Two does depend on the choice of a quota.

¹² When the square root function was proposed officially by Sweden in Nice as a way to obtain new weights from population weights, it was conceived solely as a simple method for reducing excessive advantage of large states over smaller states (the ratio of new weights is lower than that of original ones). According to Moberg “The square root was chosen in order to find a simple formula for degressivity. The resemblance to the so called ‘Penrose theory’ was a coincidence” (Moberg, 2014, p. 53). Moberg’s answer (personal communication, 2014) to my question of whether the practice of assigning weights in the past in accordance with square root proportions was raised in Nice as an argument for adopting the square root transformation as a “formula for degressivity” was negative.

Poland risked a lot, when it came up with a proposal to use square root weights to design a new voting system for the EU Council. If Germany and France accepted the proposal, Poland would have to respond with one's own concession, that is, to accept the partners' claims as to the quota. For given square root weights, represented as integers which add up to 345 (they are given in Table 8) one can construct many voting games with different properties. To dispel German doubts as to the Polish initiative, I examined a voting game obtained by setting the quota at 246. This game has a two-level blocking structure with blocking fours on the lower level (Sozański, 2007a, p. 49). The premier league consists of 16 countries (from Germany through Bulgaria). Their order with respect to the number of blocking fours reproduces the order of weights. If you compare this particular square root game with the Double Majority Game (Table 7) having the same players on the second level of the three-level irregular blocking structure, you will see that Germany's *will to power* is fulfilled to an even greater extent. The "German square root game" has a regular blocking structure and it does not need to be artificially corrected by adding the ban on blocking in threes.

Let me examine here two other square root games. The first of them, the *Jagiellonian game*, is defined by setting the quota at 213 nominal votes, which is the integer counterpart of the "optimal" relative quota of 61.6% recommended by Słomczyński and Życzkowski (2006). This game has a two-level regular blocking structure, with exclusive-hierarchical premier league made up of 8 largest states. Notice that Germany is a member of all 7 blocking fives and it has considerable advantage over other countries on the level of minimal blocking sixes.

The choice of a quota is a suitable *technical means* to achieve a given *political end*. If you set the blocking threshold r at 91 (the minimum value such that blocking in threes becomes impossible), which yields $q=255$, you will not get the "ugly" Nice game but quite a "nice" game we shall label *French game* or "French square root game." Its blocking structure very much resembles that of the Lisbon game, but it gives to Germany significantly smaller advantage over France.

The chart given in Figure 1 illustrates the distributions of relative blocking power in the two games. In a voting game having one-level blocking structure with $k_{\min}=4$, *relative blocking power* can be measured by means of index γ' defined by the formula $\gamma'(i)=b_{m,4}(i)/\sum_{m,4}(j)$.

Those who postulate that the distribution of relative power be as similar as possible to the distribution of population shares should appreciate another property of the "French square root game." Although this game has been constructed with the use of square root weights, the values of γ' are very close to the shares of the total EU population.

Table 8
The blocking structure in two games with square root weights

EU-27 member states	Square root weights	Jagiellonian game $q=213$			French game $q=255$	
		$b_{m,5}(i)$	$b_{m,6}(i)$	$\gamma(i)$	$b_{m,4}(i)$	$\gamma(i)$
1. Germany	33	7	588	89.9	174	72.8
2. France	29	6	495	75.7	137	57.3
3. UK	28	6	466	71.3	123	51.5
4. Italy	28	6	466	71.3	123	51.5
5. Spain	24	4	355	54.2	94	39.3
6. Poland	22	4	255	39.1	74	31.0
7. Romania	17	1	181	27.5	27	11.3
8. Netherlands	15	1	151	23.0	20	8.4
9. Greece	12	0	100	15.1	17	7.1
10. Portugal	12	0	100	15.1	17	7.1
11. Belgium	12	0	100	15.1	17	7.1
12. Czech R.	12	0	100	15.1	17	7.1
13. Hungary	11	0	86	13.0	13	5.4
14. Sweden	11	0	86	13.0	13	5.4
15. Austria	10	0	71	10.7	12	5.0
16. Bulgaria	10	0	71	10.7	12	5.0
17. Denmark	8	0	43	6.5	10	4.2
18. Slovakia	8	0	43	6.5	10	4.2
19. Finland	8	0	43	6.5	10	4.2
20. Ireland	7	0	33	5.0	8	3.3
21. Lithuania	7	0	33	5.0	8	3.3
22. Latvia	5	0	18	2.7	4	1.7
23. Slovenia	5	0	18	2.7	4	1.7
24. Estonia	4	0	14	2.1	3	1.3
25. Cyprus	3	0	8	1.2	3	1.3
26. Luxembourg	2	0	3	0.5	3	1.3
27. Malta	2	0	3	0.5	3	1.3
	345	7	655		239	

Since $\sum_j b_{m,4}(j) = 4b_{m,4}$, we can easily compute $\gamma'(i)$ from the data given in Table 8, by dividing $b_{m,4}(i)$ by $4b_{m,4} = 4 \cdot 239 = 956$. For Germany and France we get $\gamma'(1) = 174/956 = 18.2\%$, $\gamma'(2) = 137/956 = 14.3\%$. These values are by 1.5% greater than the relative population weights 16.7% and 12.8%. For all other countries except Romania ($\gamma'(7) = 2.8\% < 4.4\%$) and the Netherlands ($\gamma'(8) = 2.1\% < 3.3\%$), the respective difference does not exceed 1.0%

Similar calculations can be done for the Lisbon game. France's relative blocking power (13.0%) in this game is only a bit greater than this country's share of the EU population (12.8%), whereas Germany's power excess over the population share (16.7%) now equals 3.2% ($\gamma'(1) = 229/(4 \cdot 87) = 19.9\%$). Romania and the Netherlands gain again a bit less relative blocking power than their population shares. For other countries, the absolute difference between the two values never exceeds 1.0%. Thus, Germany will be the only beneficiary of the Lisbon voting system.

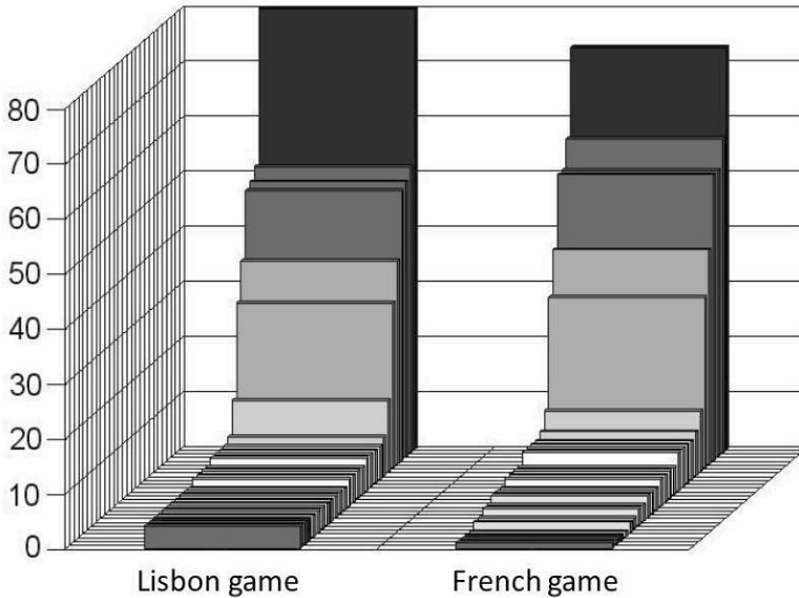


Figure 1. Relative blocking power distributions in two voting games for EU-27.

9. CONCLUSIONS AND A POSTSCRIPT

9.1. Again on politics and mathematics

Sometimes mathematical theorizing is at odds with actual “political engineering,” as it happened to the mainstream conception of voting power. Nevertheless, the mathematicians, no matter which theoretical approach they prefer, will always have *knowledge advantage* over the politicians. They are able to grasp the latter’s point of view and reorient their investigations accordingly. That’s exactly what I did, having realized that EU leaders pay little attention to the values of classical indices of winning power, trying instead to maximize the *blocking* power of their states.

There is another asymmetry. The politicians have always had *power advantage* over the mathematicians. The former can say to the latter: we do appreciate your efforts to enlighten us, but the designing of decision rules for *practical* use must remain our exclusive right. I’m not going to deny this prerogative to the EU leaders, yet in their own interest they shouldn’t ignore the voice of scientists who can often reveal some *unintended consequences* of political decisions, such as potential *instability* of the distribution of blocking power in voting games in which the rules, for being based on

external data (e.g., *current* demographic composition), may appear sensitive to small unpredictable changes in the input.

What the politicians must learn, first of all, if they are to benefit from cooperation with experts in the mathematical theory of voting games, is the distinction between strictly political issues and technical problems. The task of the politicians is to work out an agreement as to political requirements that a voting system should meet. Whether the EU leaders choose to restore the parity principle or they finally endorse Germany's claim to power advantage over other largest EU member states is a *political* issue. *How* to design a voting game which generates a given power hierarchy and meets certain formal conditions which may be no less politically important, is a *technical* problem. The "French square root game" I have constructed with the use of square root weights with the aim to bring down Germany's power advantage over France to a more reasonable size shows that *independent* experts can solve technical problems whether they do or do not approve of political ends which determine to range of solutions they have to consider.

That's how I conceive of the task of an expert, even if I realize that the *expectations* of those who need advice may affect the way in which an expert plays the expert *social role* to a greater degree than the expert's own *role conception*. The men and women of power usually expect from the men and women of knowledge to be instructed solely as to the choice of the *means*, yet at times they may welcome being instructed in the matter of *ends* as well – when they feel unable themselves to specify their objectives sufficiently clearly. However, an expert who takes up a task of the second kind must acknowledge that his advice can be dismissed and he may be blamed by his client of misrepresenting his intentions or attempting to promote one's own ideas or evaluations. As a matter of fact, many scholars love to persuade the politicians to accept both certain general principles and concrete policies the academia believes to be grounded on scientific truths. I had a similar intention when I joined (before the EU summit of June 18, 2004) the supporters of the use of square root weights in constructing a voting system for the EU Council.

9.2. Which way forward?

Three years later two initial signatories of the *Letter to the Governments*, professors Jesús Mario Bilbao (University of Sevilla) and Karol Życzkowski (Jagiellonian University) were given an opportunity to defend Penrose's theory once again at the debate which took place May 23, 2007 at the European Policy Center in Brussels. I will state here my own answer to the question "Which way forward?" which was asked there. Today (November 2014) I would direct to the EU leaders and *their* experts the same suggestions which I formulated in a letter to Bilbao and Życzkowski

before the debate in question. These suggestions are repeated below as they seem to me no less reasonable than 7 years ago.

- Regardless of whether you do or do not accept the *classical conception of voting power*, you should appreciate square roots weights as a convenient *means* for designing voting games.
- Represent square root weights in a fixed integer form in order to be able to determine exact numbers of small blocking coalitions. The integer weights may add up to the number given in the Nice treaty (345) or a comparable number.
- Simplify the allocation of weights by dividing the set of players into groups that are assigned the same weight. Here political decisions have to be made. If you choose to finally part with the *parity principle* in relation to the Big Four, tell it overtly to the world instead of hypocritically invoking the principle of democratic representation to justify the decision to make Germany the strongest player.
- For given political weights obtained by adjusting original square root weights, try various quotas until you manage to construct a voting game with blocking structure that will be both regular and politically acceptable for all EU member states.
- If you can't find a better solution, implement the Jagiellonian voting game which is well constructed and should be politically acceptable too, for it gives enough blocking power to Germany – the leader of the “premier league” in this game. If you believe that the EU needs further integration and reaching positive decisions should be made easier, then you should not object to raising from 4 to 5 the minimum size of any “blocking minority.”
- You can add the 1 state–1 vote game with a proper quota (say, 15 countries) if the winning threshold you have chosen for the game with square root weights can be attained by some coalitions formed by less states than a simple (minimum) majority. The set of *small* blocking coalitions will not change, but smaller countries will be more comfortable with a system whose egalitarian component has a stronger effect on forming *winning* coalitions.

9.3. Postscript

9.3.1. The treaty reforming the institutional structure of the European Union was signed by 27 governments (Romania and Bulgaria joined the EU on January 1, 2007) December 13, 2007 in Lisbon. At long last, the opponents gave in, but they insisted on leaving the Nice treaty voting system in force as long as possible to the effect that the transition to the new voting system was deferred for 7 years. Croatia, which became the 28th member of the EU on July 1, 2013, had an opportunity to act as

a member of the Council under the old voting rules for a rather short time, as the Lisbon voting game would come into force November 1, 2014. The new member state, for its population a little smaller than Denmark, Finland, Ireland, and Slovakia and greater than Lithuania, got as many votes (7) as each of these 5 countries. As a consequence, the total number of nominal votes in game G_1 (see 4.3) rose from 345 to 352. The new quota for this game was raised from 255 to 260 in accordance with the rule, introduced by the constructors of G_1 , that the minimum total weight of a blocking coalition must always equal 91. The quota in G_2 was set at 15 to ensure that the second component of the Nice game remains a *simple* majority game. The third component, or the population game G_3 , didn't need to be modified, as the quota for this game (62% of the total EU population) was defined independently of the number of member states. The Nice voting game, for all its deficiencies, was quite well adjusted to the admission of new players.

Among many questions concerning the effect of Croatia's accession on power distribution, there is one that appears most interesting in the context of this paper, namely, one would like to know if the blocking structure of Nice game has changed anyhow after the last EU enlargement. In particular, is the Berlin-Paris-Madrid axis still in a position to block any decision of the Council? If you use the data, published by Eurostat in the form of exact numbers representing the population distribution on January 1, 2013,¹³ you will get again 4 blocking coalitions of size 3 (same as those shown in 4.3). As previously, {France, Italy, UK} cannot block Germany, although its share of the whole EU population has risen from 36.9% in 2007 to 37.1% in 2013. Interestingly, the coalition {Germany, UK, Spain}, for which the share of the total EU-28 population now equals 37.985% is now very close to attaining the blocking threshold of 38%.

Felsenthal and Machover (2009, p. 319) noticed that the "the new QM rule [the Lisbon voting system] is the first in the history of the EU whose functioning can be affected *significantly* by changes in population size." They also pointed to the lack of "legally binding procedure" that would allow to obtain the population size for each country "at synchronized regular intervals." Actually, the EU leaders had already in Nice agreed on a procedure of collective decision-making that takes as input an external parameter whose distribution is sensitive to fluctuations beyond the actors' control. They failed (did they try?) to construct a voting game, similar to that used in the EU-15, with weights defined so as to render acceptable differences in *political status* between member states and to give to the whole structure certain desirable *formal properties*. Instead, they chose the weakest (however preferred by the strongest player) component of Nice treaty game for the cornerstone of the Lisbon game.

¹³ Germany: 82020578, France: 65578819, UK: 63896071, Italy: 59685227, Spain: 46727890, Poland: 38533299, etc. the total EU-28 population being equal to 507162571.

9.3.2. The specialists in the theory of voting games who had been interested in the interplay with the world of politics welcomed the appending to the Lisbon treaty of a clause on the prolonged use of the Nice system. They were given more time for carrying out further analyses of the existing voting systems and studying alternative institutional solutions. Felsenthal and Machover (2009) presented their predictions (based on demographic forecasts reaching up to 2060) as to the evolution of the voting power distribution over few decades to come. Since they assumed that there would be no enlargement of the EU-27, the accession of Croatia diminished practical importance of their calculations. In their analyses, these scholars devoted some space to how blocking opportunity varies in voting systems with varying population weights. Their approach falls under classical way of theorizing which prevails in the field until today, as evidenced by the collection of papers *Power, Voting, and Voting Power: 30 Years After*, which appeared in 2013. Holler and Nurmi, the volume editors, recalled in their paper (2013, p. 4–7) the competition, far from being resolved, between the analysts who consider the Banzhaf index the “right index” of voting power and those who like more the Shapley-Shubik index. At the end, the authors consented to Robert Aumann’s opinion on different concepts of a “solution” in game theory; all of them “depict or illuminate the situation from different angles; each one stresses certain aspects at the expense of others.”

The same can be said on a special aspect of the functioning of voting systems which is called *blocking power*. This kind of power can be roughly defined as the ability to prevent others from achieving their goals attainable only through collective action subject to certain rules. Blocking power depends on the system’s structure and may be absent in some assemblies that take decisions by voting or it can be evenly or unevenly distributed across the members of such bodies. In this paper – instead of staying with Coleman’s index of “preventive power”, a measure which has been preferred by many theorists who seem to have forgotten or neglect the original meaning of the term *blocking coalition* – I used the old definition of the latter term as a starting point for a new path within the mathematical theory of voting games. The approach proposed here leads to defining few other concepts and offers some new ways of quantifying blocking power. I believe that it can be further developed so as to provide sociologists and political scientists (I mean those who are open to “mathematical thinking”) with more tools for analyzing structural global and local properties of the systems they study.

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