SCHELLING GAMES, KURAN DOMINOS AND ELECTORAL COALITIONS. NON-STANDARD GAME-THEORETIC MODELS OF COLLECTIVE ACTION[1]

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Abstract: *Non-cooperative games such as the Prisoner's Dilemma, Stag Hunt, Asymmetric Coordination and others are primary tools used for modeling collective action. I consider formal models that are close cousins of such standard games: Schelling's games, Kuran's dominos and partition function form games. For certain empirical problems, each of these formalisms may have advantages over standard games. Among the benefits there are mathematical simplicity, more intuitive depiction of represented phenomena, and better operationalizability. I formalize all three models and prove simple existence theorems for two of them. The detailed examples of applications include vaccination, unpredictability of revolutions, and electoral coalitions.*

Key words: *collective action, non-cooperative games, partition function, Schelling's games, Kuran's games, Mancur Olson.*

1. Introduction

Collective action has proven to be one of the most important concepts in the social sciences, fertilizing the research of sociologists, political scientists, economists and others. Mancur Olson initially analyzed collective action using customary economic tools (Olson 1965, especially Chapter 1) but non-cooperative game theory quickly became the standard for dealing with collective dilemmas. Arguably the most popular representation of collective action is the Prisoner's Dilemma (PD), developed by Flood and Dresher (Flood 1952) and made famous by Tucker's (1950) story. In that game,

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dominant strategies produce an outcome that is Pareto-dominated by the outcome resulting from dominated strategies.

Some researchers have argued that other games represent collective action at least as well as the PD and its multi-player and multi-strategy extensions. In Volunteer's Dilemma (Poundstone 1992: 201-4), players have weakly dominant strategies that lead to an outcome that is weakly Pareto dominated by another outcome (think about soldiers facing a live grenade falling into their trench: who would sacrifice himself to save the others?). Games such as Stag Hunt (Assurance) or Asymmetric Coordination, and their generalizations, do not even have weakly dominant strategies (see Skyrms 2004 for a comprehensive discussion of Stag Hunt). In such games one of the Nash equilibria is Pareto-dominated by another equilibrium or a non-equilibrium outcome. If, for empirical reasons, the players are initially stuck or more likely to end up in an inefficient equilibrium, we get dilemmas that match our intuition of what constitutes collective action. Game-theoretic models of collective action go far beyond the 2x2 games. Hardin (1982) and Sandler (1992) provide good overviews.

The goal of this paper is to review three formal models of collective action outside of the traditional realm of non-cooperative game theory, present rich (non-stylized!) applications for each case, and prove basic results. The three different types of models discussed below have certain comparative advantages over standard non-cooperative games. Schelling's games model interactions that non-cooperative game theory would struggle to represent due to inherent measure-theoretic difficulties in games with a continuum of players. Kuran's domino games could be in principle translated into a non-cooperative framework but his representation is both simpler and more intuitive. Both types of games as well as the concept of equilibrium are formalized, and the existence of equilibria is established. Finally, partition function-form games deal with coalition formation, a type of interactions that non-cooperative games are hopelessly unsuccessful in modeling. All models are illustrated with detailed applications to vaccination, revolutions and the formation of electoral coalitions.

2. Schelling's games of binary choice

Let's start with a simple formalization of Schelling's games that are applicable to collective action. This mode of modeling appeared in Thomas Schelling's early analyses and was developed with many insightful examples in Schelling (1973). In applications of game theory to biology, similar models are often analyzed as "population games" (see, e.g., Milchtaich 2014).

A *Schelling's game* is defined as a pair of two continuous functions $c(x)$ and $d(x)$: $[0,1] \to \mathbf{R}$. A *Schelling's equilibrium* (or simply equilibrium) $x \in [0,1]$ is a single point x satisfying one of the following conditions, depending on whether $x = 0$, $x = 1$ or whether $x \in (0, 1)$:

(1) for $x \in (0, 1)$: $c(x) = d(x)$;

(2) for $x = 0$: $d(0) \geq c(0)$;

(3) for $x = 1$: $c(1) \geq d(1)$.

Functions c and d are interpreted as the payoffs resulting from the two strategies in the game, cooperation (C) and defection (D), and the argument x is interpreted as the proportion of cooperators. According to definition, x is an equilibrium when the payoffs from cooperation and defection are identical, when defection is no worse than cooperation and nobody cooperates, or when cooperation is no worse than defection and everybody cooperates. When the payoffs from both strategies are equal, the most interesting case is when an equilibrium is stable. In such an equilibrium, a small perturbation in the proportion of cooperators x provides incentives to players to change their strategies in such a way that x becomes closer the equilibrium.

The following existence result guarantees that Schelling equilibria always exist (see the Appendix for proofs):

Proposition 1: Every Schelling's game has at least one equilibrium.

Unless we are interested in absolute values of payoffs, the function $\Delta(x) = c(x) - d(x)$ conveys all information that is necessary for analysis and, in particular, for the calculation of equilibrium. This also means that we can normalize either of the two functions defining Schelling's game in order to obtain a particularly convenient form of the game.

One of the original examples that Schelling used to illustrate his graphs is the smallpox vaccination dilemma (Schelling, 1978: 222-4). It is assumed that the optimal number for vaccination is 90 percent of the population. Thus, those who are vaccinated bear some additional cost of mild allergic reactions and even the small risk of occasional death versus no-risk-all-benefits of non-vaccinators. Schelling's question is: 'Who should be vaccinated?' He ingeniously speculates about a system determining who would be vaccinated, including possibly 'fractional vaccination.'

Today's bureaucratized health providers and Big Pharma lobbies fail to consider any sophisticated fair schemes for vaccination. Current vaccination philosophy assumes that every child should get inoculated. However, the vaccination dilemma has re-emerged recently in a novel way. The 2015 Disneyland outbreak of measles is the tip of the iceberg in the United States and Europe for this seemingly eradicated disease.[2] An increasing number of parents believe that vaccination would harm their children and refuse to immunize them despite no reliable research confirming their fears. The vast majority of those immunized is outraged. The small minority often invokes their religious rights but in fact they are afraid of potential side-effects.

How can we make sense of this curious phenomenon? If we try to model vaccination as a simple strategic game, we will either get buried under the complexity of our model or won't get any sensible results. In this case, a Schelling's model can be very helpful.

Let's assume that vaccination reduces the risk of catching the disease to zero and that the cost of vaccination (fees, invested time and side-effects) is constant regardless of the number of participants. Thus, the payoff from being vaccinated is constant and let's normalize it at zero. The payoff from defection increases with the proportion of population that is vaccinated. It starts below the payoff from vaccination, i.e., below zero, and at some sufficiently high proportion of immunized, it reaches zero. Then it increases further to reach a positive value (see Figure 1).

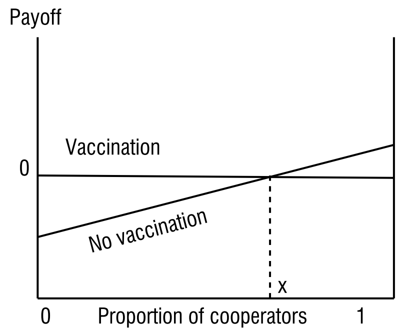

***Figure 1*. Vaccination Game. The 'Vaccination' and 'No vaccination' lines denote the payoffs derived from playing the corresponding strategies; *x* denotes the equilibrium proportion**

The dynamics in the Vaccination game represents the following intuition: when nobody is vaccinated, a player expects to benefit from the vaccine by acquiring

[2] Physicians seem to be unprepared for returns of 'eradicated' diseases. In anecdotal evidence, the author (who lives in Orange County near Disneyland), got a measles-like rash at the time of the outbreak. The kind urgent care physician hesitated about what to do, spent some time on the phone consulting, and ordered \$500+ of tests. After getting home, the author called a physician in Poland, where measles was eradicated later than in the United States and where there are still specialists well-familiar with the disease. The grumpy doctor asked three simple questions, laughed at the battery of tests, made a sarcastic comment about bureaucratic American medicine, and offered a definite diagnosis: not the slightest chance of measles. The next days confirmed her diagnosis. Over the next weeks, the author got three calls from different Orange County specialists with three diagnoses (1) tests inconclusive but probably not measles; (2) weak form of measles; (3) ignore the previous call, no measles.

immunity to disease at a small cost of his time, fees paid to healthcare provider and expected side effects. When the proportion of players with immunity increases, the probability of contracting the disease decreases while the costs remain constant. When the proportion of immunized is high enough to make the probability of contracting the disease sufficiently small, the costs are equal to the benefits. Finally, when almost everybody is vaccinated, the costs outweigh the benefits since the non-immunized enjoy the benefit of protection without the costs of immunization. (If you were the last person on Earth who was not vaccinated against some human borne disease, from whom could you catch it?)

There is a unique stable Schelling equilibrium in the Vaccination game, namely, the point x_0 when both payoff functions cross. Analyzing this simple equilibrium can give us a surprisingly rich interpretation. While introducing vaccines is without a doubt a huge health improvement that saves uncountable lives, a point must be reached when non-vaccination becomes a better strategy. The popular perception of such a saturation point may be obviously biased in many ways, but its very existence is sufficient to motivate popular backlash against vaccination.

The justification for not vaccinating may come from flawed research, religious beliefs or the fear of the evil Big Pharma. However, the real mechanism that supports the strategy of non-vaccination is the lack of punishment imposed on non-vaccinators. The benefits from playing both strategies when *almost* everybody – but not *literally* everybody – cooperates are virtually identical. The further we are from the equilibrium, the greater the gap between cooperation and non-cooperation that either motivates the non-cooperators to vaccinate or the cooperating majority to start defecting. Near the equilibrium, the insignificant gains or losses from vaccination may fall below the recognition level of a typical player regardless of what she does and whether she understands the accounting of actual benefits.[3]

Interestingly, some parents and physicians understand the intuition behind the above argument and base their vital decisions on it. Dr. Rob Sears, an Orange County pediatrician, agrees when his anxious patients do not want to vaccinate their offspring. He doesn't make any of the wild claims typically associated with non-vaccination. He doesn't claim that any treatments alternative to jabs work better. He acknowledges that with no vaccination, many apparently eradicated diseases would return. His only claim is that the strategy of "hiding in the herd" – the herd being the vaccination jargon for the vaccinated – may be quite safe only because the vast majority of children are vaccinated.[4]

[3] Notice that in Vaccination it is reasonable to assume that payoffs are not just ordinal but that they are better measurable, so the concept of 'small differences' between payoffs is meaningful.

[4] This paragraph describes the content of and cites Lexington (2015).

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In an interesting twist of events, Schelling's system for fair vaccination emerged as a weird de-centralized social process with forces that keep vaccination rates near the equilibrium. While the equilibrium is not necessarily the 'optimal' proportion of vaccinated, it is reasonable to assume that it is close enough for typical game parameters. A potential source of problems is that the equilibrating forces may be not precise enough to guarantee that the deviations from equilibrium are negligibly small.

Finally, let's notice that strategic games, especially with a continuum of players, have serious disadvantages when used to model collective action problems like vaccination. Schelling's games get around the mathematical difficulties by implicitly disposing of the concept of player and focusing only on the proportion of cooperators.[5]

3. Kuran's domino games of surprising revolutions

Mancur Olson (1990: 9) described mass dissatisfaction with authoritarian regimes as a classical problem of collective action:

> When there are no free elections and governments are able and willing to use force, political outcomes do not mainly depend on the hearts and the minds of the people – on what the various populations and nationalities want. Very often governments can be massively unpopular yet continue in power … The logic of collective action keeps the huge number of people who don't like a regime from taking action that would overthrow it.

While certainly fitting very well within the domain of collective action, the distaste for a regime has one incredibly interesting feature that sets it apart. Revolutions – solutions to the problem of mass dissatisfaction with rulers in authoritarian regimes – tend to happen suddenly and surprisingly. Or they do not happen when they are expected. The dynamics of forming a revolution resembles chain reaction. This

[5] A simple 2x2 game doesn't capture the spirit of the dilemma sufficiently well. A better way to represent players would be to use a continuum [0, 1]. However, the inherent measure-theoretic difficulties associated with continuum of players produce paradoxical results. Since it is necessary to define the 'proportion' of all players playing the same strategy, one must introduce a concept of σ-algebra and measure on all subsets of the continuum. Under all non-atomic measures, there is always an infinite set of players who could change their strategies without changing other players' payoffs. Furthermore, some subsets of players (not measurable) wouldn't be 'allowed' to play one strategy versus all remaining players playing the opposite strategy. While one could try to go around such problems, the resulting formalism would be complex.

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striking phenomenon is well-recognized in mass culture and often brings fascinating and powerful depictions of revolutions.

Andrzej Wajda's 1981 Oscar-nominated and Palm d'Or winning movie 'Man of Iron' tells a story of a factory worker fighting the communist oppression in Poland. Communist censorship apparently removed from that movie a powerful scene that takes place in a huge power facility in northern Poland. The use of power gradually decreases. When the intrigued engineer consults the giant map of manufacturing facilities, he sees how one bulb after another shuts down. This means that the facility represented by the bulb has stopped drawing power. Then the camera shows how workers are gradually joining the strike and power down their machines.

Another well-known movie scene comes from the Wachowski's siblings 'V for Vendetta'. V, the anarchist revolutionary, plays with dominoes arranged close to each other by slightly tilting the first piece. The piece hits the next one, which hits another one, and the chain reaction of revolution unfolds. In the background, one nasty incident with a regime functionary sparks a parallel chain reaction. Angry activists snowball into a revolutionary crowd destroying the autocratic regime.

Both scenes nicely illustrate the main idea of Kuran's (1989) seminal article. Kuran in his perfectly timed paper published in March 1989 was arguably the first one to use a formal 'domino model' to argue why revolutions are often unpredictable and why we are surprised when they happen. His inspiration was the Islamic revolution in Iran of 1979. Surprisingly, just a few months later a wave of revolutions in Central Europe swept down apparently ironclad Soviet-imposed communist regimes! The first semi-free elections held in Poland on June 4, 1989, led to the formation of a non-communist cabinet. Hungary was soon next with a communist-free cabinet. In November, the Berlin Wall fell. Czechoslovakia, Bulgaria, and in December Romania, followed East Germany closely. The regimes fell like domino pieces, in a clear sequence of events, and the dynamics within the regimes unfolded according to a chain-reaction scenario. Two years later, the mighty Soviet Union itself became a footnote in a history book. More recently, a wave of surprising 2011 'Arab Spring' revolutions overthrew rulers in Tunisia, Egypt, Libya and Yemen, and caused major uprisings and protests in eight more countries in North Africa and Middle East.

Unlike the other examples discussed in this paper, Kuran's model may be easily re-defined as a strategic game. However, Kuran's formal representation conveys his ideas more succinctly and intuitively. I will describe informally the underlying non-cooperative structure and then convert it into Kuran's framework.

Let's assume for simplicity that we have $n \geq 3$ players. Each player is fully described by his *threshold*, a non-negative integer, which is interpreted as the minimal number

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of other players who join the revolution that motivates this player to join. The player's threshold may range from zero (we may interpret such a player as a brave *dissident*) to n (a *dictator* who never joins the revolution against himself). Between the dissident and dictator player attitudes may vary from courageous supporters (threshold 1) to opportunists (threshold n-1).

An n-player *Kuran's domino* is any vector of $K = (k_i)_{i=0,\dots,n}$ of $n+1$ non-negative integers, where $\Sigma_i k_i = n$, that state the numbers of players with thresholds 0, 1, …, n, respectively, starting with the player(s) with the smallest threshold(s). Alternatively and more conveniently for definitions and proofs, we can represent a Kuran's domino as a cumulative distribution of K, i.e., as $n+1$ non-negative integers $(N_i)_{i=0,\dots,n}$ such that $N_i = \Sigma_{j\leq i} k_j$. Thus, $0 \leq N_0 \leq N_1 \leq \dots \leq N_n = n$. We interpret N_i as the number of players with thresholds not greater than i. An *equilibrium* in a Kuran's domino is any non-negative integer i such that for $i = 0$, we have $N_0 = 0$ or for $i > 0$, we have $N_{i-1} = i = N_i$. The first case represents a situation when nobody joins since all players need at least one other player joining. The second condition represents the case of precisely i players joining the revolution since all their thresholds are below i ($N_{i-1} = i$) while there are no additional players with threshold i who would also like to join ($N_i = i$).

An equilibrium i in a Kuran's domino is thus defined as the number of players who joined the revolution in such a way that no participant wants to stop participating and no non-participant wants to join. The equilibrium describes only the number of players who join the revolution rather than the entire player strategy profile that would appear in a non-cooperative game. The following proposition holds:

Proposition 2: Every Kuran's domino has at least one equilibrium.

The appeal of Kuran's domino lies in how it helps us to visualize the 'domino effect' and to make surprising predictions about the inevitability of surprise. Let's assume for simplicity that we have ten players and our game is described by the following eleven numbers representing the numbers of players with thresholds 0, 1, 2, …, 10, respectively:

Game K_1: (1, 0, 2, 1, 1, 1, 1, 1, 1, 0, 1).

We have two Kuran's equilibria: $i_1 = 1$ and $i_2 = 9$. The first equilibrium may be interpreted as a society with a single dissident. In the second equilibrium, everybody joins the revolution except for the dictator. This is the starting point for analyzing what happens when there is a truly minuscule change in the society. Let's say that the threshold of one player goes down from 2 to 1. We obtain a game that is only slightly different from the original K_1:

Game K_2: (1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1).

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Unlike in K_1, $i_1 = 1$ is no longer an equilibrium. With our dissident present, the now emboldened player with threshold 1 joins as well. But with two players joining, the player with threshold 2 has incentive to join. And so on: all players 1-9 join one after another. The only equilibrium in K_2 is when nine players join. We have a chain-reaction revolution.

The difference between K_1 and K_2 is tiny. If the initial state of affairs in a society was just a small opposition group embodied by the dissident, then the spark that triggered the process of eight players joining the revolution included only the slight lowering of one player's threshold. The question now is: can we detect such a potentially eruptive situation and predict that under some preference patterns in a society revolutions are imminent?

Unfortunately, identifying dormant revolutions is problematic. We cannot resort to public opinion polls since in autocratic societies they are notoriously inaccurate. Even with a truth-seeking autocrat, surveys will be likely contaminated by high refusal rates, strategic misrepresentation of respondent preferences, and respondent ignorance about political alternatives (Kaminski 1999). Studying preferences in autocracies is prone to systematic mistakes. Kuran's model offers us the reason why predicting revolutionary outcomes is difficult even if we can correctly identify pre-revolutionary ferment. In the next game, all the thresholds equal to 3-8 go down to just 2:

Game K_3: (1, 0, 8, 0, 0, 0, 0, 0, 0, 0, 1).

This is a massive change from the situation described in K_1. The societal majority got seriously radicalized. However, as massive as this change is, it doesn't spark a revolution. All eight players with thresholds 2 need two more players with lower thresholds in order to join. But there is only one player with such a threshold! We still have two equilibria $i_1 = 1$ and $i_2 = 9$. In contrast, a tiny change leading to game K_2 started such reaction by annihilating the first equilibrium. Thus, in a society whose revolutionary fever is already pretty difficult to diagnose, one small change may start a revolution while a much more massive change won't. The inherent sensitivity of revolutionary chain reaction to small changes of parameters while at the same time being insensitive to apparently much bigger changes makes revolutions so unpredictable and surprising!

In general, Kuran's games cannot be represented as Schelling's models since implicit players are no longer identical. However, one can construct another model with a continuum of players extending the Kuran's game in the following way: A game is defined as a continuous and non-decreasing reaction function $r(x)$ on [0, 1] such that $r(0) \geq 0$ and $r(1) \leq 1$. The reaction function at level x tells us the proportion of population that wants to join the revolution given that the population observes that x of them joined. I leave it to the reader to complete the description and play with such a model.

4. Partition-function form games of electoral coalition formation

Our third example of a non-standard model of collective action is motivated by the process of coalition formation and uses a generalization of characteristic function form games.

Consider the dilemma of parties in a multi-party system getting ready for elections. They must decide whether to form an electoral coalition with other parties (we will not distinguish between tactical temporary coalitions and permanent mergers), whether to split into smaller pieces, or do nothing. Note that the post-election dilemma of forming a cabinet coalition is relatively simpler since the parties' payoffs (seats) are additive, and there is no uncertainty about seat shares. In contrast, an electoral coalition of parties may receive fewer or more votes than what the coalition members would obtain when competing separately. The totals of votes in both cases depend on the electorate preferences. To complicate calculating even more, when the electoral law translates votes into seats, it may (and typically does) reward larger parties with a bonus. Furthermore, the payoff of a party or a coalition depends also on whether other parties created their own coalitions. A framework that models similarly complex dilemmas was introduced by Thrall (1962) as partition function form, or PFF, games (see also Rosenthal 1972, Chwe 1994, and Kaminski 2006 for generalizations of the PFF framework).

A PFF game (P, v) includes a set $P = \{P_1, P_2, \ldots, P_n\}$ of at least three parties and a *payoff function* v. A *coalition* K is any nonempty subset $K_i \subseteq P$, including singletons. A *coalitional structure* S is any exhaustive family of at least two disjoint coalitions $\{K_1, \ldots, K_r\}$, i.e., any partition of P that includes at least two elements. The payoff of $K_i \in S$ is denoted as $v_S(K_i)$. Since we interpret payoffs as percentages of seats, we assume that for all structures S, $v_S(K_i) \geq 0$ for all $K_i \in S$ and $\Sigma_i v_S(K_i) = 100$ (seat percentages are non-negative and all seats are allocated). Note that only the payoffs of coalitions from a given structure are defined; we have no data on the payoffs of coalition's members.

Without delving too deeply into further formalism, let's assume that an equilibrium in a PFF game is a coalitional structure such that no coalition has incentive to split or further merge. Thus, no coalition can split into smaller pieces in such a way that its members would receive jointly more seats by competing separately; also, no two or more coalitions can increase their total seat share by further coalescing. We can measure the 'degree of instability' of a coalitional structure S, Δ_S, as the largest potential gain that the coalitions in S could capture via splitting or merging. With some caution we can interpret Δ_S as the extent to which the collective action problem

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was not solved by a subset of coalitions.[6] In equilibrium, $\Delta_S = 0$. There are games with no equilibrium:

Proposition 3: Every three-player game has a split-merger stable equilibrium. For every n>3, there exist n-player PFF games with no equilibrium.

Collective action problems typically arise in electoral coalition formation due to *fragmentation*, i.e., when small parties fail to coalesce – often dramatically so – even though further consolidation would give them more seats. One of the most spectacular fragmentations took place in Central European elections soon after the fall of communism. The plankton of rightist parties of Lithuania (in 1992), Poland (1993) and Hungary (1994) failed to create unified blocs. This failure led to a 'redshift' in Central European politics, i.e., the unexpected electoral success of ex-communist parties that enjoyed increased support due to reform backlash but that were far from receiving the majority of votes (see Kaminski et al., 1998).

In Poland in 1993, the six rightist parties received jointly 26.2% of votes and only 3.5% of seats. Had they coalesced, their seat shares would have likely been between 32% and 38.3% (Kaminski et al., 1998: 442). Several factors that contributed to their collective failure included the unpredicted shift in support shortly before the election, the last-minute entry of a new competitor (Solidarity trade union) just before the deadline for registering electoral committees, the inability of public opinion polls to adequately capture changes in support, and the Chicken-like structure of pre-electoral coalition formation games.

Estimating even a small part of a partition function is usually a complex endeavor and requires the help of an appropriately designed public opinion survey. Kaminski (2001) obtains such estimates for the coalitional structure of Polish party system in 1998 that followed the fragmentation of 1993 and the resulting long consolidation. While the coalitional structure that gradually formed over four years after 1993 was not in equilibrium, it was substantially more stable than the original one. The degree of instability was estimated to go down from 31.6 in 1993 to 12.2 in 1997.[7]

The consolidation process that solved the collective action problem of fragmentation was extremely complex. Figure 2 reconstructs the most important components of this process, i.e., major splits, mergers, coalitions, entries of new parties, coalitional negotiations, defections and withdrawals from elections between 1993 and 1997. Needless to say, no non-cooperative game would be able to reasonably capture any crucial details of this process and turn any useful predictions or explanations. The

[6] Caution in interpretation must be maintained because there exist games with no equilibrium. In such games, the lowest value of Δ_S is greater then zero.

[7] Note that Proposition 3 tells us that it is possible that there was no equilibrium in the actual game. The empirical estimates reconstructed only a small part of the partition function and were not sufficient to decide whether an equilibrium coalitional structure existed.

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PFF-based model combined with survey data generated an estimate of the instability of the party system, predictions that were soon fulfilled by subsequent coalitional adjustments and, as an unexpected side-product, discouraged the biggest party in Poland from coalescing with a smaller rival (see Kaminski 2001 for details).

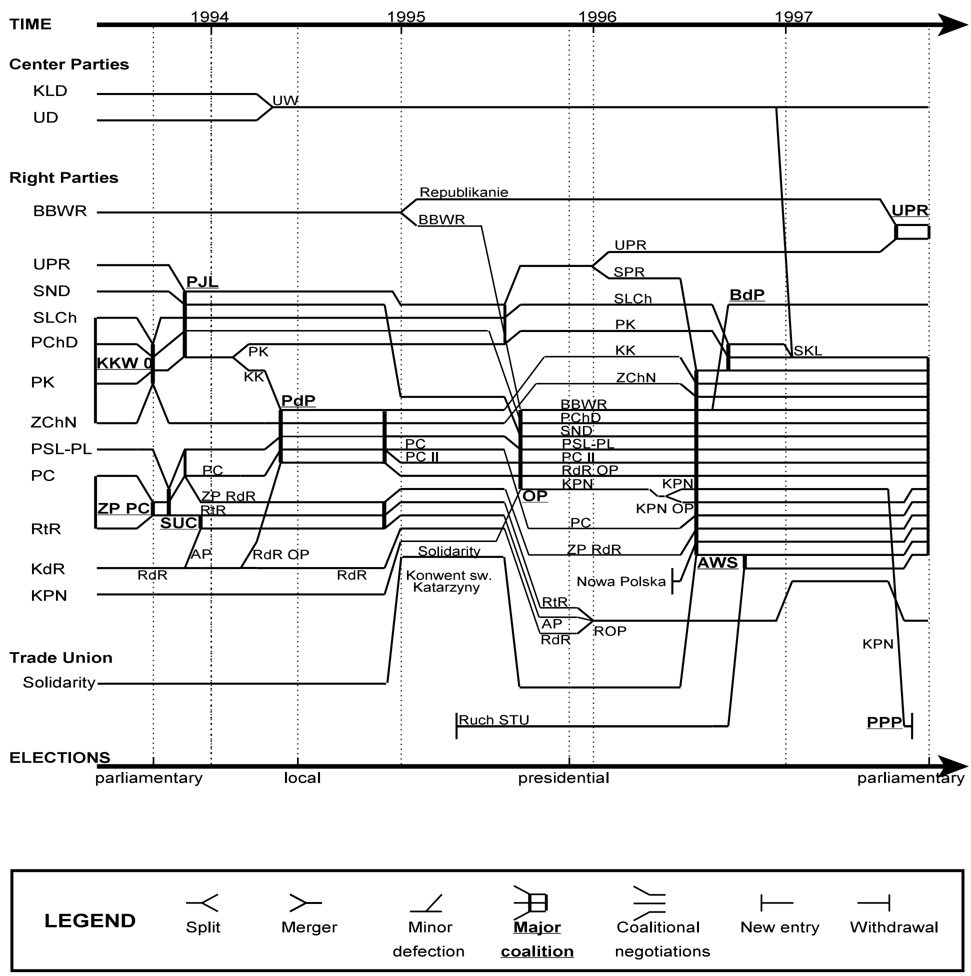

Figure 2. **A solution to the collective action problem of 1993 fragmentation of the rightist parties in Poland**

Notes: Every line represents the story of coalitional, split, merger, etc. activity of a non-ephemeral political party or coalition on the Right or Center. Minor players, minor changes of players' identities or names, and players other than political parties except for the Solidarity trade union are omitted. Dates are approximate. Relative player strength is not shown. Vertical positions of players do not represent their spatial positions within the Center-Right cluster.

Source: Kaminski 2001.

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5. Conclusion

Mancur Olson's (1965) notion of collective action helped to conceptualize and analyze a great number of empirical situations. The argument here is that while non-cooperative game theory (including one-shot strategic games, repeated games or complex extensive form games) often provides the most natural framework for modeling collective action, we have other tools at our disposal as well. The models discussed in the paper are close relatives of standard strategic games. Their distinct advantages in some situations lie in their simplicity and the avoidance of certain problems associated with noncooperative games. Schelling's games go around the mathematical difficulties associated with the continuum of players. Kuran's games allow for a more intuitive representation of a latent chain reaction underlying dormant revolutions or similar processes. Both types of models can be also represented as certain dynamical systems. Partition-function form games are different and apply tools from cooperative game theory. They focus on potential gains from coalescing that drive coalition formation and entirely ignore the hardly tractable process of negotiations.

A general point can be made. Nash (1951) suggested a program for game theory to convert all cooperative games, as introduced by von Neumann and Morgenstern (1944), into non-cooperative games. Von Neumann created cooperative games since he saw difficulties in using non-cooperative framework for all games and problems. The ambitious 'Nash program' proved to be very fruitful and led to many spectacular results in game theory and its applications. Without questioning the insights and benefits provided by the Nash program, one has to be aware of limitations built into the standard non-cooperative framework even in application to mainstream political phenomena. The models considered in this paper highlight such limitations and motivate the quest for other formal representations.

6. Appendix

Proof of Proposition 1: Consider function $\Delta(x) = c(x) - d(x)$ defined on [0,1]. By continuity of $c(x)$ and $d(x)$, $\Delta(x)$ is continuous and by compactness of its domain, it achieves its minimum m and maximum M. The simple proof below evaluates three exhaustive and exclusive cases:

(1) $m \geq 0$: This means that $\Delta(x) \geq 0$ for all $x \in [0,1]$. Thus, $c(1) \geq d(1)$, and by definition, 1 is an equilibrium with full cooperation;

(2) $M \leq 0$: By a similar argument, 0 is an equilibrium with full defection;

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(3) m and M have different signs: By continuity, $\Delta(x)$ has Darboux property, i.e., for some $x_0 \in (0,1)$, $\Delta(x_0) = 0$, and x_0 is an equilibrium. n

Proof of Proposition 2: Consider a Kuran's domino $K = (n_i)_{i=0,\ldots,n}$. Two cases are possible:

(a) $N_0 = 0$. In such a case 0 is an equilibrium by the first part of our definition of equilibrium;

(b) $N_0 > 0$. Let's assume that K has no equilibrium. In such a case $N_1 > 1$ since otherwise $N_0 > 0$ and $N_1 = 1$ would imply that $N_0 = 1$, and that 1 is an equilibrium. By induction, $N_i > i$ for $i = 2, \ldots, n$, what is a contradiction since $N_n = n$.

Proof of Proposition 3: See Kaminski (2001), proofs of Propositions 1 and 2.

References

Chwe, Michael. (1994). Farsighted coalitional stability. *Journal of Economic Theory, 63*, 299-325.

Flood, Merrill M. (1952). *Some Experimental Games*. *Research Memorandum RM-789*. Santa Monica, CA: RAND Corporation.

Hardin, Russell. (1982). *Collective Action*. Baltimore: Johns Hopkins University Press.

Kaminski, Marek M. (1999). How Communism Could Have Been Saved. Formal Analysis of Electoral Bargaining in Poland in 1989. *Public Choice, 1-2*(98), 83-109.

Kaminski, Marek M. (2001). Coalitional Stability of Multi-Party Systems. *American Journal of Political Science, 45*(2), 294-312.

Kaminski, Marek M. (2006). General Equilibrium Model of Multiparty Competition, *Social Choice and Welfare, 26*(2), 336-361.

Kaminski, Marek M., Grzegorz Lissowski and Piotr Swistak. (1998). The "Revival of Communism" or the Effect of Institutions? The 1993 Polish Parliamentary Elections. *Public Choice, 97*(3), 429-449.

Kuran, Timur. (1989). Sparks and prairie fires: A theory of unanticipated political revolution. *Public Choice, 61*(1), 41-74.

Lexington. (2015). Thundering herd. *The Economist, 26 (September)*, 35.

Milchtaich, Igal. (2014). *Static Stability in Symmetric and Population Games*. Working paper, Bar-Ilan University econ.biu.ac.il/files/economics/working-papers/2008-04_0.pdf (accessed on November 5, 2015).

Nash, John. (1951). Noncooperative games. *Annals of Mathematics, 54*, 289-295.

Olson, Mancur. (1965). *The Logic of Collective Action: Public Goods and the Theory of Groups*. Cambridge, MA: Harvard University Press.

Olson, Mancur. (1982). *The Rise and Decline of Nations: Economic Growth, Stagflation, and Social Rigidities*. New Haven: Yale University Press.

 DOI: 10.7206/DEC.1733-0092.62

Olson, Mancur. (1990). The logic of collective action in Soviet-type societies. *Journal of Soviet Nationalities, 1*(2), 8-33.

Poundstone, William. (1992). *Prisoner's Dilemma*. Anchor Books: New York.

Rosenthal, Robert. (1972). Cooperative games in effectiveness form. *Journal of Economic Theory, 5*, 88-101.

Sandler, Todd. (1992). *Collective Action. Theory and Applications*. Ann Arbor: University of Michigan Press.

Schelling, Thomas C. (1973). Hockey Helmets, Concealed Weapons, and Daylight Saving: A Study of Binary Choices with Externalities. *Journal of Conflict Resolution, 3* (17), 381-428.

Skyrms, Brian. (2004). *The Stag Hunt and the Evolution of Social Structure*. Cambridge: Cambridge University Press.

Thrall, Robert M. (1962). Generalized Characteristic Functions for n-person Games. In: *Recent Advances in Game Theory*. Princeton: Princeton University Conferences, 157-160.

Tucker, Albert W. (1950). *A two-person dilemma*. Mimeographed paper, Stanford University.

von Neumann, John and Oskar Morgenstern. (1944). *Theory of Games and Economic Behavior*. Princeton: Princeton University Press (reprinted 1990).

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