THE EFFECT OF DELAY ON RISK TOLERANCE AND PROBABILITY WEIGHTS[1]

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Abstract: *People often face choices where outcomes are both delayed and uncertain. Numerous studies of delayed gains and losses show the hyperbola-like discounting process of delayed payoffs. We assumed that when evaluating a delayed lottery people act according to the prospect theory model, where payoffs are discounted according to a hyperbolic function. The problem that we addressed is whether the α parameter of the value function v( ) and the γ parameter of the probability weighting function w( ) differ for evaluations of delayed and instant lotteries. We found that when people compare delayed certain payoffs with delayed risky payoffs they are more risk prone than in situations where both certain payoffs and risky payoffs are instant. However, when people compare present certain payoffs with delayed risky payoffs they are less risk prone than in situations where both certain payoffs and risky payoffs are instant. Additionally,*

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the probability weighting curves were more linear for delayed lotteries than for instant lotteries (people were more sensitive to changes of probability).

Key words: *time discounting, probability discounting, delayed lotteries, hyperbolic discounting, prospect theory.*

WPŁYW ODROCZENIA WYPŁAT NA SKŁONNOŚĆ DO RYZYKA ORAZ NA WAGI DECYZYJNE

Streszczenie: *Ludzie często stają w obliczu wyborów, których efekty są zarówno odroczone, jak i niepewne. Liczne badania odroczonych zysków i strat pokazują, że są one dyskontowane w czasie według funkcji hiperbolicznej. W niniejszym badaniu zakładamy, że przy wycenie odroczonych loterii ludzie będą postępować zgodnie z modelem teorii perspektywy, w którym wypłaty są dyskontowane w czasie według funkcji hiperbolicznej. Problem, który sobie postawiliśmy, jest następujący: czy przy loteriach odroczonych w czasie zmienia się stosunek jednostki do ryzyka? Okazało się, że gdy ludzie porównują odroczone, pewne wypłaty z odroczonymi ryzykownymi wypłatami, są bardziej skłonni do ryzyka (wartość parametru α rośnie), niż w sytuacji, gdy zarówno pewne wypłaty, jak i ryzykowne są natychmiastowe. Jednak gdy ludzie porównują natychmiastowe pewne wypłaty z odroczonymi ryzykownymi wypłatami, są mniej skłonni do ryzyka (wartość parametru γ maleje), niż w sytuacji, gdy zarówno pewne wypłaty, jak i ryzykowne są natychmiastowe. Dodatkowo funkcja wag decyzyjnych okazała się bardziej liniowa dla odroczonych loterii, niż w przypadku loterii natychmiastowych (ludzie są bardziej wrażliwi na zmiany prawdopodobieństwa w przypadku odroczonych loterii).*

Słowa kluczowe: *dyskontowanie w funkcji czasu, dyskontowanie w funkcji prawdopodobieństwa, odroczone loterie, hiperboliczne dyskontowanie, teoria perspektywy.*

1. INTRODUCTION

People often choose between actions where the consequences of these actions are uncertain. The most popular theory of decision making under risk and uncertainty is Kahneman and Tversky's (1979) prospect theory. According to this theory, the overall value of an uncertain/risky option is given by the sum of the subjective values of outcomes multiplied by the decision weights associated with the probability of the

outcomes. For a simple risky prospect-like lottery L = (A, p) with only one non-zero payoff A and probability p associated with it, the subjective value of a lottery V is:

$$V = v(A) \cdot w(p), \qquad [1]$$

where $v(\ )$ is the value function and $w(\ )$ is the probability weighting function.

Another class of decision situations concerns delayed consequences. Human decision making across instant and delayed payoffs was formally described by Mazur (1987). Mazur's hyperbolic discounting function is represented by the following formula:

$$V = \frac{A}{1+kD}, \qquad [2]$$

where: V is the present subjective value of the future payoff, A is the amount of future payoff, D is the delay time and k is a discounting coefficient.

However, in real life people often face choices where outcomes are both delayed and uncertain (e.g., in natural catastrophes potential losses are not only uncertain but also delayed). In the present research the combined effect of risk and delay was studied. A crucial question centers upon how uncertain and delayed gains are discounted. This topic was studied by Vanderveldt, Green, and Myerson (2015) who proposed the following formula for the evaluation of this type of option:

$$V = \frac{A}{(1+kD)(1+h\frac{1-p}{p})}, \qquad [3]$$

where $1+kD$ refers to a time discounting hyperboloid function, $1+h\frac{1-p}{p}$ refers to a probability discounting hyperboloid function, $\frac{1-p}{p}$ expresses the chances against a particular event occurring ("odds against").

In practice Vanderveldt et al. use the formula in Equation 4, where s_d and s_p are exponent characteristics of time and probability discounting respectively. Adding these exponents to the formula (Equation 4) improves the fit to empirical data:

$$V = \frac{A}{(1+kD)^{s_d}(1+h\frac{1-p}{p})^{s_p}}. \qquad [4]$$

Equation 4 assumes that both delayed and uncertain payoffs are discounted according to hyperboloid functions. Vanderveldt et al. conducted an experiment in which participants made choices between smaller, certain and immediate rewards and larger, uncertain, delayed rewards. They found that Equation 4 fitted experimental

data well. Yet, we consider this formula not to be fully satisfactory. A series of research studies, which will be presented below, shows that people may reveal different risk attitudes for instant and delayed lotteries. Equation [4] does not seem useful enough for studying such effects. Therefore in the present research we decided to use a different approach based on Mazur's equation [2] and prospect theory. We compared the quality of fit to empirical data for both approaches ([4] and [5]).

We assumed that Formula 1 could be used, replacing instant rewards with future rewards discounted to the present, this led to the following equation:

$$V = v(\frac{A}{1+kD}) \cdot w(p).$$ [5]

The value function for gains (where payoffs are higher than 0) may be represented by the power value function:

$$v(A) = A^{\alpha}.$$ [6]

The parameter α represents risk tolerance, i.e., for α between 0 and 1 the value function represents risk aversion, for $\alpha > 1$ risk seeking, and for $\alpha = 1$ risk neutrality.

This results in the following formula:

$$V = (\frac{A}{1+kD})^{\alpha} \cdot w(p).$$ [7]

A problem remains as to whether the parameters of the value function $v(\ )$ and the probability weighting function $w(\ )$ change with the delay. In previous research it was found that when choosing between two delayed lotteries people are more risk tolerant than in the case of two instant lotteries. In experiments conducted by Noussair and Wu (2006) participants compared two lotteries with payoffs materializing either in the present or in the future. The authors found that when participants compared lotteries with payoffs in the future they were less risk averse than for instant lotteries. Similarly, Abdellaoui, Diecidue, and Öncüler (2011) examined certainty equivalents for both instant and delayed lotteries. It should be emphasized that certainty equivalents for delayed lotteries were also expressed as future payoffs. It was found that certainty equivalents for delayed lotteries were bigger than for instant lotteries. Thus, again, participants were more risk tolerant for lotteries with future payoffs than for instant lotteries.
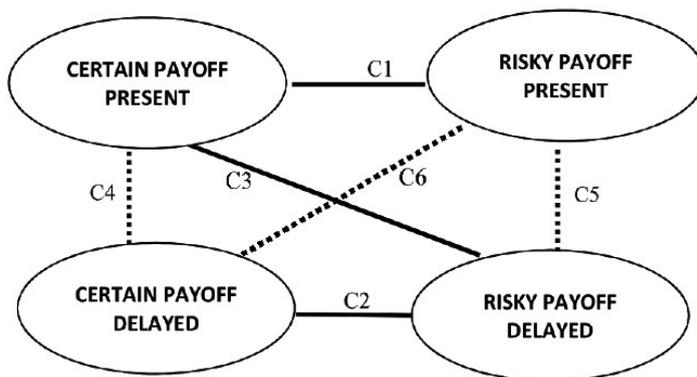
A characteristic of the choice tasks in the above experiments was that participants were comparing risky options with certain (or less risky) payoffs occurring at the same time, either immediately or in the future. A comparable real life situation may be where a customer has a choice between two restaurants, one where food is always of medium quality, and one where food can be either delicious or really bad. Here, the experimental test would concern differences in choice between the two restaurants

for a present dinner and for a future dinner (for instance after three months). As already said, the results of such experiments indicate that if outcomes occur instantly people are less risk prone than when they occur in the future.

However, there are situations where people have to evaluate delayed risky options in terms of their present value. For instance, a plaintiff claiming damages may consider whether they should accept an instant indemnity proposed by a defendant or wait for a higher but uncertain indemnity resulting from a court decision. Similarly, a farmer may have a choice of whether to sign an advance agreement with a firm purchasing corn at a fixed price or whether to take the risk of waiting (and buy an option). What might be expected for these types of comparison?

Let us start by clarifying the evaluation process when an individual compares risky vs. certain and present vs. delayed payoffs. As shown in Figure 1 by three solid lines, we can ask participants about:

1) the present certainty equivalent of an instant lottery – in this case the participant compares the certain present payoff vs. a risky present payoff (C1),

2) the delayed certainty equivalent of a delayed lottery – in this case the participant compares the certain delayed payoff vs. a risky delayed payoff (C2),

3) the present certainty equivalent of a delayed lottery – in this case the participant compares the certain present payoff vs. a risky delayed payoff (C3).

Figure 1. **Three types of comparison involving risky vs. certain payoffs, and present vs. delayed payoffs used in our research**

In the first situation (C1), when an individual is asked to compare the instant lottery with the present certainty equivalent, they have to compare something uncertain (uncertain gain) with something certain (certain gain), both in the present. Here, in line with Equation 1, when evaluating a lottery, according to prospect theory an individual combines the value function and the probability weighting function.

The same evaluation process takes place in situation C2, where both an uncertain gain and a certainty equivalent concern the future. Since both the certain and uncertain gain involve the same time frame, the individual does not have to take into account time discounting. Thus, again according to prospect theory (Equation 1), when evaluating a lottery an individual combines the value function and the probability weighting function. However, risk attitude for present and future prospects does not have to be the same. The alpha coefficient in the value function for future prospects can differ from that for present prospects. Indeed, research on discounting certain gains vs. discounting lotteries shows that individuals reveal less steep time discounting for lotteries as compared with certain payoffs (Ahlbrecht & Weber, 1997). Thus, if we delay both a lottery and a sure reward, a lottery will gain in attractiveness in comparison to a sure reward. As a consequence of this, people will be more risk seeking when options involve the future than when options involve the present; this accords with experiments by Noussair and Wu (2006) and Abdellaoui et al. (2011).

Now let us look at the third comparison in Figure 1 (C3), where an individual is asked to compare a delayed lottery with the present certainty equivalent. This situation involves comparing options that differ in two dimensions simultaneously: (1) time discounting and (2) probability discounting. Thus, when evaluating a distant lottery in terms of a present certainty equivalent an individual has to compare a certain and instant payoff with a prospect comprising two sources of uncertainty, requiring probability and time discounting. Accepting the claim of Rachlin et.al. (1991) that probability and delay discounting functions have the same general shape, an individual can consider the delayed lottery as a type of compound risky prospect.

To our knowledge there have been only two studies (Blackburn & El-Deredy, 2013 and Vanderveldt, Green, & Myerson, 2015) where this type of comparison has been studied. However, there have been studies on the violation of the so-called reducibility of compound lotteries axiom. According to this axiom, the rational decision maker should be indifferent between a compound lottery and the equivalent simple lottery. An equivalent simple lottery is a lottery containing the same outcomes as the compound lottery and probabilities which are equal to multiplication of the respective probabilities of the compound lottery. For example, the simple lottery "you win a dollar with 25 percent probability" is equivalent to the compound lottery: "you win a dollar if a coin toss comes up heads and then a die rolls an even number".

Contrary to this axiom, research shows that when choosing between equivalent lotteries people prefer one-stage over multi-stage lotteries (Budescu & Fischer, 2001; Halevy, 2007). Dillenberger (2001) and Abdellaoui, Klibanoff, and Placido (2015) even introduced the concept of compound risk aversion, meaning that the decision maker's certainty equivalent for a compound lottery is below the certainty equivalent for a simple lottery. Spears (2013) showed that some people prefer simple over compound lotteries even when the simple lotteries offer lower expected value. Perhaps people consider prospects comprising more sources of uncertainty as less attractive than those comprising only one source of uncertainty.

By analogy, when evaluating a distant lottery in terms of a present certainty equivalent, an individual may consider prospects comprising (1) time discounting and (2) probability discounting as extremely unattractive. Compared to the certain and instant payoff, the delayed lottery may look unappealing, therefore the decision maker will exhibit higher risk aversion to delayed lotteries than to instant lotteries.

In line with the above considerations we formed the following hypotheses:

- *H1. When people compare delayed certain payoffs with delayed risky payoffs they are more risk prone than when they compare present certain payoffs with present risky payoffs. Operationally, this means that when people compare delayed certain payoffs with delayed risky payoffs, the alpha parameter in equation [7] is larger than when they compare present certain payoffs with present risky payoffs.*

- *H2. When people compare present certain payoffs with delayed risky payoffs they are less risk prone than when they compare present certain payoffs with present risky payoffs. Operationally, this means that when people compare present certain payoffs with delayed risky payoffs, the alpha parameter in equation [7] is smaller than when they compare present certain payoffs with present risky payoffs.*

Finally, there is the question of whether the delay of a lottery affects the probability weighting function. Abdellaoui et al. (2011) found that the probability weighting function becomes more linear as delay increases. The authors claimed that the effect is due to anticipated emotions which are probably less intense for delayed lotteries than for instant lotteries. They referred to findings of Rottenstreich and Hsee (2001) who reported that people tend to be less sensitive to probability change when they react to affect-laden objects. Following the same reasoning we hypothesize that:

- *H3. In delayed lotteries the probability weighting curves will be more linear (people will be more sensitive to changes of probability) than in imminent lotteries. Operationally, this means that when people compare delayed certain payoffs with delayed risky payoffs or present certain payoffs with delayed risky*

payoffs, the gamma parameter in equation [10] is larger than in imminent lotteries.

To test our hypotheses we conducted two experiments. In both experiments we elicited certainty equivalents (indifference points) for each lottery. However, different reference points were used in each experiment. In the first experiment participants compared lotteries with certain payoffs occurring at the same time, either immediately or in the future (comparisons C1 and C2 on Figure 1). In experiment two, participants compared either immediate or delayed lotteries with certain payoffs occurring in the present (comparisons C1 and C3 on Figure 1). Having median equivalents for each lottery, we could estimate the parameters of the value and probability weighting functions for each delay separately.

Figure 1 also shows possible comparisons which were not examined in our research. C4 and C5 show options that differ on only one dimension – time discounting: C4 shows a certain present payoff vs. a certain delayed payoff (a standard delay discounting task), and C5 depicts a present risky payoff vs. a delayed risky payoff. Finally, C6 shows a delayed certain payoff vs. an instant risky payoff.

2. EXPERIMENT 1: CONCURRENT CERTAINTY EQUIVALENTS FOR PRESENT AND DELAYED LOTTERIES

Method

Participants. Participants were 57 students recruited from Warsaw University of Life Sciences (27 females, 30 males, mean age = 19.5 years). Data for 5 participants (1 female) was excluded from analysis because they gave more than three non-consistent responses (a maximum of one non-monotonic response was allowed inside each delay, e.g., if the indifference point for a 75% chance of receiving 100 experimental currency units was higher than for a 95% chance this was treated as erroneous).

Materials and procedure. The experiment consisted of one computerized session. There were 15 rounds. In each round participants faced a series of choices between a lottery and a sure payoff which both materialized at the same time. The rounds were grouped into three parts which corresponded to one of three delays: 0, 3 or 12 months. For each delay, participants were presented five lotteries with the same amounts to win (3000 PLN[6] or nothing), but which differed with respect to probability of winning: 5%, 25%, 50%, 75%, 95%. Within each delay, the lotteries were presented in a random order to avoid any order effects.

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[6]   It is approximately $800.

The experiment took about half an hour. Participants were invited to the computer laboratory and each one sat at a separate computer. The experimenter explained that the purpose of the study was to examine preferences for different amounts of money. Participants were also informed that there were no wrong choices. Before participants started the task the experimenter gave a deeper explanation about the options to be chosen from. He explained that the payoff could be certain or could be risky, i.e., occur with a specified probability. Payoffs could also be available immediately or after some delay. Finally, it was explained that some payoffs would be a combination of two attributes: they could occur both with some probability and after some delay.

Students were informed that after the experiment took place one randomly chosen person would receive a monetary reward for participation in the research, i.e., they would get money according to their performance in the experiment. It was explained that this meant that one of their decisions would be randomly selected and implemented for real. It was emphasized that the payment would be made on the specific date (immediately, or in 3 or 12 months) as described in the particular decision problem.

At the beginning of the experiment, participants read the instruction:

"Our university organizes a lottery twice a year: in the winter (*January*) and in the autumn (*October*). Imagine that you have a ticket for the lottery. You can use it in one of two ways:

A. You can take part in A LOTTERY, which will be resolved IN SOME DEFINED TIME and, if you win, you will receive the payoff immediately after drawing, but, if you lose, you will get nothing.

B. You can change the lottery ticket for a voucher worth SOME AMOUNT OF MONEY, which you will receive at the same time as the lottery takes place."[7]

Then they took part in 15 rounds of choice situations. Before making a decision task in each round participants were asked to consider the details of the lottery:

- the amount of money to win,
- the probability of winning,
- the time of drawing and payment.

This process was used to make participants more conscious about the risky and delayed options which they would consider. Then participants made several choices between the presented lotteries and a series of sure payoffs. Experimental fillers were used after each part of the experiment (participants were asked to observe two pictures of cats or dogs for 15 seconds and had to make a hypothetical choice as to which of them they would like to take home for a week).
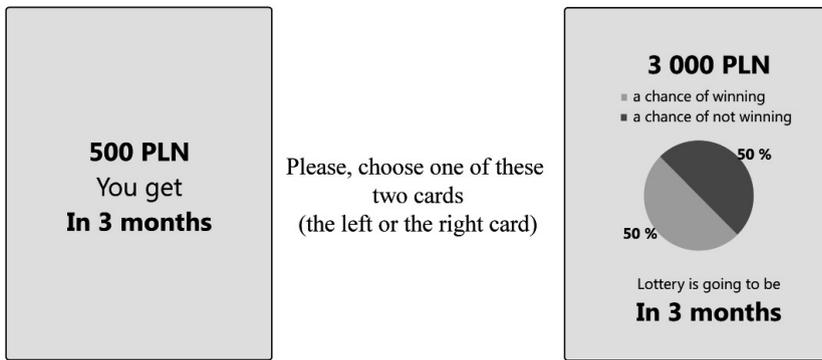
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[7]   In Experiment 2 the expression "at the same time as the lottery takes place" was replaced with the expression "immediately".

Elicitation of certainty equivalents (adjusting procedure)

A procedure was used to elicit certainty equivalents (indifference points) for 15 lotteries. The elicitation procedure was based on an *adjusting method* similar to that used in previous studies of discounting (e.g. Du, Green, & Myerson, 2002). (For review of different measures of time preferences see Hardisty, Thompson, Krantz, & Weber, 2013.)

In each round, after participants had familiarized themselves with the lottery they saw two cards on the computer screen: the left card displayed the amount and time of a sure payment and the right card displayed the lottery and time of its realization (Figure 2).

Figure 2. **Two options that a participant chose between**

During the adjusting procedure the amount on the right card was constant, whereas the amount on the left card changed according to participants' previous choice. In the initial choice the amount of the sure payment presented on the left card was half of the value of the gain in the lottery (i.e., 1500 PLN). The participant chose between these two options by clicking on one of the two cards. When a participant chose the delayed and risky option (the right card) the certain payment on the left card increased by half of its previous value (i.e., 2250 PLN). When a participant chose the safe option (the left card) the certain payment decreased by half of its previous value (i.e., 750 PLN). This algorithm was repeated eight times and in the last step the computer program calculated the indifference point for the risky and delayed option.

Results

First, we calculated median values of certainty equivalents for five levels of probability values and three delays.

Table 1
Median certainty equivalents for five different probability values and three delays

Probability	now	3 months	12 months
0.05	117	170	246
0.25	451	474	380
0.50	750	1060	1201
0.75	1406	1494	1570
0.95	2080	2261	2385

A Friedman non-parametric test was conducted for each probability level to determine whether certainty equivalents differentiated among three delays. Results of that analysis indicated that there was a significant difference in median certainty equivalents among three delays for risky payoff with a probability value of 0.05, $\chi2(2, N=52)=10.60$, $p=0.005$ and there was a statistical tendency in the differences between risky payoffs with a probability value of 0.50, $\chi2(2, N=52)=5.93$, $p=0.051$.

Table 1 shows that the certainty equivalents:

- increased as the probability of a gain increased;
- increased as the delay increased (although not all of the differences in certainty equivalents between two different delays were statistically significant, and in two cases the sign of the differences was the opposite to that predicted).

The first result is consistent with the basic proposition that the higher the probability of a gain becomes, the higher is the value of an indifference point. The second result confirms Hypothesis 1.

Additionally, we examined two types of monotonicity of certainty equivalents for each participant separately. In the first type of monotonicity certainty equivalents are expected to increase with increases in the probability of a gain for each delay. We performed 12 comparisons of certainty equivalents for each participant and found that on average 10.6 responses were consistent with the first type of monotonicity. In the second type of monotonicity certainty equivalents are expected to increase with increases in the delay for the same probability of a gain. We performed 10 comparisons of certainty equivalents for each participant and found that on average 6.5 responses were consistent with this second type of monotonicity.

We calculated the parameters of the value function and probability weighting function both at the individual level and for the aggregate data. However, estimates obtained from individual data were very noisy. As noted by Cohen, Sanborn, and Shiffrin (2008), this effect is probably due to the very small number of observations: estimation of the parameters for each delay was based on only five lotteries and their certainty equivalents. We therefore present only the median parameters of the value function and probability weighting function, estimated separately for each delay.

To estimate the parameters of the value function and probability weighting function Formula 5 was fitted to group median certainty equivalents for risky payoffs across the five probabilities using non-linear least squares regression analysis. The certainty equivalent is an amount at which a person is indifferent between receiving it for sure or playing a lottery. This means that the utility of this equivalent is the same as the overall utility of a risky option. Consequently, using Formula 5 we obtained the following equation:

$$v(\frac{CE^c}{1+kD}) = v(\frac{A}{1+kD})w(p), \qquad [8]$$

where CE^c is the concurrent certainty equivalent for a lottery (the present certainty equivalent for an immediate lottery, and the future certainty equivalent for a delayed lottery).

In this experiment the effect of delay on the subjective value of the sure or risky payoff is omitted because payoffs were paid at the same time. To estimate the above equation we assumed a power value function:

$$v(x) = x^\alpha, \quad for \quad x > 0. \qquad [9]$$

and used a one-parameter exponential form of the probability weighting function suggested by Prelec (1998):

$$w(p) = e^{-(-\ln p)^\gamma}. \qquad [10]$$

So, we performed nonlinear least squares regression for the following equation:

$$CE^c = A(e^{-(-\ln p)^{\gamma_d}})^{1/\alpha_d}. \qquad [11]$$

We fitted the above equation to median certainty equivalents to estimate parameters α_d and γ_d (for each delay D separately). The parameter α usually lies in the range [0,1] and generates a concave value function. The lower the parameter α is the more the value function represents risk averse behavior. The γ parameter is assumed to be in the range [0,1] and generates the inverse S-shape probability weighting function. It reflects sensitivity to differences in probabilities. The nearer to 1 parameter γ is the more linear the weighting function is, representing a greater ability to discriminate between probabilities.

Table 1 presents estimates of parameters α_d and γ_d for group median data. As can be seen from Table 1 and Figure 3 (right panel), the values of parameter α_d increase with the delay. In consequence, the value function is more linear for delayed than for instant lotteries and indicates that participants were more risk tolerant for delayed lotteries relative to instant lotteries. Thus, H1 was supported. It should be noted, however, that our model was fitted to aggregate choices (median data), not to individual responses, therefore we are not in a position to test the significance of differences between parameters for various delays.
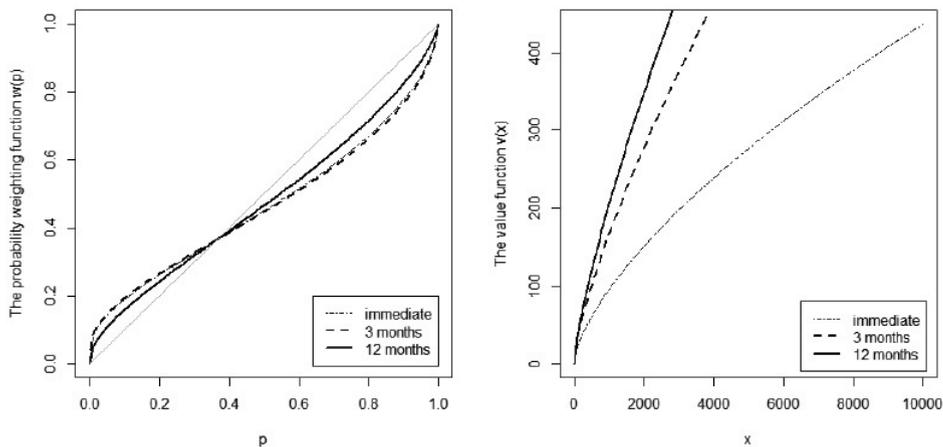
As can be seen from Table 2, the curvature parameter γ_d of the probability weighting function increases with the delay. Figure 3 (left panel) shows that the probability weighting function is more linear for delayed than for instant lotteries. Thus, H2 was supported; sensitivity to changes in probability was higher for delayed lotteries than for instant lotteries.

Table 2
Parameter estimates for the probability weighting function and the value function calculated separately for different delays for median data (N = 52) in Experiment 1

Parameter*	now	3 months	12 months
γ_d	0.61	0.59	0.73
α_d	0.66	0.74	0.77

*All γ_d parameters are significant at $p < .01$, all α_d parameters are significant at $p < .001$. $R^2 = 0.98$.

Figure 3. **The probability weighting function (left) and the value function (right) at each delay in Experiment 1**

3. EXPERIMENT 2: PRESENT CERTAINTY EQUIVALENTS FOR IMMEDIATE AND DELAYED LOTTERIES

Method

Participants. Participants were 56 students recruited from Warsaw University of Life Sciences (38 females, 18 males, mean age = 19.5 years). Similarly to Experiment 1, one randomly selected participant could win an amount of money depending upon their choice for a randomly selected decision. Data for 8 participants (4 females) was excluded from analysis using the same elimination procedure as in Experiment 1.

Materials and procedure. The procedure was similar to that used in Experiment 1 with two exceptions: (1) the sure payoffs (on the left card) in each choice situation always occurred in the present, (2) two extra rounds elicited indifference points for 3 or 12 months for sure payoffs of 3000 PLN.

Results

We calculated median values of certainty equivalents for five levels of probability values and three delays.

Table 3
Median certainty equivalents for five different probability values and three delays

Probability	Now	3 months	12 months
0.05	175	99	99
0.25	369	187	205
0.50	796	744	615
0.75	1166	1253	1230
0.95	2250	2630	2361

A Friedman non-parametric test was conducted for each probability level to determine whether certainty equivalents differentiated among three delays. Results of that analysis indicated that there was a significant differences in median certainty equivalents among three delays for risky payoff with probability value of 0.05, $\chi2(2, N=48)=12.63$, $p=0.002$ and for risky payoff with probability value of 0.25, $\chi2(2, N=48)=7.45$, $p=0.024$. Moreover, there was a statistical tendency in differences for risky payoffs with probability value of 0.50, $\chi2(2, N=48)=5.42$, $p=0.067$.

Table 3 shows that the certainty equivalents for Experiment 2:

- increased as the probability of a gain increased;
- decreased as the delay increased, but only for lower probability values; certainty equivalents for delayed lotteries did not change for 0.75 and 0.95 probabilities.

The first result is consistent with the basic proposition that the higher the probability of a gain becomes, the higher is the value of an indifference point. The second result is consistent with the discounting rule, i.e., delayed payoffs (including lotteries) are generally less preferred than the same instant payoffs (including lotteries), so the certainty equivalent for a delayed lottery is expected to be lower than that for the same instant lottery. However, note that there was little difference in certainty equivalents for delayed gains with high probabilities (0.75 and 0.95).

The above said, in the present case a simple comparison of present and delayed certainty equivalents did not allow us to test Hypothesis 2 because the delayed certainty equivalents measured both participants' risk attitudes and their discounting of delayed gains.

Additionally, we examined the monotonicity of certainty equivalents for each participant separately. We found that certainty equivalents increased with increases in the probability of a gain for each delay. For 12 certainty equivalent comparisons we found that on average 10.6 responses were consistent with the first type of monotonicity. We also found that certainty equivalents decreased with increases in delay for the same probability of a gain. For 10 comparisons of certainty equivalents for each participant on average 6.5 responses were consistent with the second type of monotonicity. These results indicate that certainty equivalents generally met the assumptions of monotonicity and that participants gave reliable responses.

Similarly to Experiment 1, we present only the median parameters of the value function and probability weighting function, estimated separately for each delay.

Using the group median equivalents for two delayed and certain payoffs, it was possible to calculate the temporal discounting factor k. For this purpose we used Mazur's hyperbolic discounting function (Equation 2) and performed nonlinear least squares regression. We obtained $k = 0.012$ ($R^2 = 0.997$). This estimated value of k is not far from values obtained by other researchers (Frederick, Loewenstein, & O'Donoghue, 2002).

In the second step, factor k was taken into account when the parameters of the value and probability weighting functions were estimated. We used Formula 5 and assumed the same parametric specifications of the value and the probability weighting

functions as in Experiment 1. We then performed nonlinear least squares regression for the following equation:

$$CE = \frac{A}{1+kD}(e^{-(-\ln p)^{\gamma_d}})^{1/\alpha_d}, \qquad [12]$$

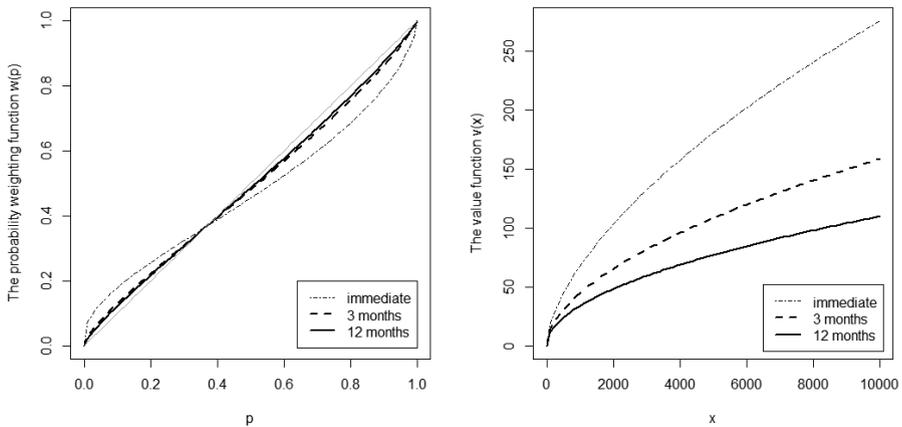where CE is the present certainty equivalent for a lottery.

Table 2 presents estimates of parameters γ_d and α_d for group median data. As can be seen in the table and in Figure 4, the values of parameter α_d decrease with the delay. Consequently, the curve of the value function is more concave for delayed than for instant lotteries. Thus, H2 was supported; participants were less risk tolerant for delayed lotteries than for instant lotteries.

At the same time, the curvature parameter γ_d of the probability weighting function was higher for instant lotteries than for delayed lotteries. This results in a more linear probability weighting function curve for delayed lotteries compared to instant lotteries (see Figure 4, left panel). Thus, as in Experiment 1, H2 was supported; sensitivity to changes in probability was higher for delayed lotteries than for instant lotteries.

Table 4
Parameter estimates for the probability weighting function and the value function calculated separately for different delays for median data (N=48)

Parameter*	Now	3 months	12 months
γ_d	0.65	0.85	0.89
α_d	0.61	0.55	0.51

*All γ_d parameters are significant at p < .01, all α_d parameters are significant at p < .001. R^2=0.99.

Figure 4. **The probability weighting function (left) and the value function (right) at each delay in Experiment 2**

4. Discussion

Previous research (for a review, see: McKerchar, Green, & Myerson, 2010) showed that a hyperboloid function provides a good description of delayed rewards discounting. Vanderveldt, Green, and Meyerson (2015) extended this model to a situation in which rewards were both delayed and probabilistic. These authors showed that a model consisting of a multiplicative combination of delay and probability hyperboloid discounting functions (see Equation 4) fitted their data well ($R^2 > 0.99$). Our real rewards study revealed two things. First, the model proposed by Vanderveldt et al. (2015) provides a good fit to our data ($R^2 = 0.99$)[8]. Second, Equation 5 provides just as good a fit ($R^2 = 0.99$), based on a hyperboloid function of delayed rewards and the prospect theory model for risky choices. Perhaps this result is not surprising. As noted by Blackburn and El-Dereedy (2013), non-linear regression R^2 is not suitable for assessing goodness of fit. Still, our formula has an obvious advantage over Vanderveldt, Green, and Meyerson's model. Our model (Equation 5), as well as including parameter k, also allows estimation of parameter α of the value function $v(\ )$ and parameter γ of the probability weighting function $w(\ )$. Moreover, we can observe how they change with delay.

Consistent with previous research (Noussair & Wu, 2006; Abdellaoui et al., 2011), the results of our first experiment showed that when individuals have to choose between delayed certain and delayed risky payoffs they tend to be more risk tolerant than when both certain and risky payoffs are instant. Operationally, this means that the certain instant equivalent of an instant lottery is smaller than the certain delayed equivalent of the same delayed lottery. The underlying mechanism for this change in risk attitude is not completely clear. An obvious explanation is that discounting rates for sure amounts are steeper than those for risky prospects. Alternatively, some authors (Noussair & Wu, 2006) attribute this effect to the fact that people discount future payments so heavily that discounted outcomes of a future lottery are perceived as very small. Thus, while people may be reluctant to take a risk when a lottery takes place immediately, they may accept the same lottery when it is postponed and its stakes are perceived as much smaller. On the other hand, according to other researchers (Abdellaoui et al., 2011) a delay has no impact on subjective valuations of outcomes. Rather, a delay makes people more optimistic about their chances of winning.

So, depending on a researcher's point of view, the increase in risk tolerance accompanying an increase in a lottery's temporal distance can have its roots in either a change of the utility function or a change in the probability weighting function.

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[8]   This is calculated for a version of Equation 4 with three parameters (excluding s_D). An equation with 4 parameters failed to converge and provided parameter k and s_D estimates that did not differ significantly from 0.

Our study showed that both functions are affected. The alpha parameter becomes larger and the probability weighting function becomes more linear. The results of Experiment 1 did not provide us with a definitive argument as to which of these two factors is decisive for the change in risk attitude.

As far as Experiment 2 is concerned, we cannot establish a change in an individual's risk attitude through a direct comparison of the certainty equivalents of the instant and delayed lotteries as was the case in Experiment 1. This is because the delayed certainty equivalents included both participants' risk attitudes and their discounting of delayed gains. So, we were forced to compare alpha parameters in value functions when both a lottery payoff and a certain payoff were instant and when an individual chose between a delayed risky payoff and an instant certain payoff. We found that median alphas were smaller in the second case than in the first case. We interpret this as showing a higher tolerance for risk in the second case.

Subsequent to conducting our studies we came across a working paper by Rolison et al. (2013). Similarly to our experiments, participants in their experiments were asked to compare:

(1) a lottery and a certain payoff, both instant, or

(2) a lottery and a certain payoff, both delayed, or

(3) a delayed lottery and an instant certain payoff.

Additionally, a fourth group compared an instant lottery and a certain delayed payoff.

In general, Rolison et al. obtained results that were consistent with ours. The only exception was that they did not confirm the common finding that when people compare delayed certain payoffs with delayed risky payoffs, they are more risk prone than when they compare instant certain payoffs with instant risky payoffs. However, more importantly, the authors emphatically replicated our Experiment 2 findings showing that alpha parameters in value functions are higher when both a lottery payoff and a certain payoff are instant than when participants compare a delayed lottery and an instant certain payoff. Thus, similarly to our experiment, when people compared present certain payoffs with delayed risky payoffs they seemed to be less risk prone.

The comparison between a delayed certain payoff vs. an instant lottery and an instant certain payoff vs. a delayed lottery was particularly interesting: people showed a propensity for risk when they compared an instant lottery with a delayed certain payoff. On the other hand, people showed an intolerance for risk when they compared a delayed lottery with an instant certain payoff. This may imply that time discounting looms larger than uncertainty discounting in human decision making.

How to explain, why people comparing present certain payoffs with delayed risky payoffs are less risk tolerant than when they compare present certain payoffs with present risky payoffs. We suggest that this result is caused by lack of tolerance for conjunction of two types of uncertainty expressed by two different discounting processes: time discounting and uncertainty discounting. This can be considered analogous to compound risk aversion: prospects comprising more sources of uncertainty look less attractive than those comprising only one source of uncertainty (Dillenberger, 2001; Abdellaoui et al., 2015). There is also an alternative interpretation: lack of tolerance for conjunction of time discounting and uncertainty discounting could be caused by negative affect associated with the prolonged state of uncertainty – the delayed state of uncertainty could be extremely aversive. For example, this interpretation may describe the behavior of consumers purchasing products on the Internet who are allowed to return a product within 14 days. Casual observation shows that consumers rarely use this option. This could be due to anticipation of an unpleasant time waiting for a few weeks without knowing whether the complaint has been accepted.

Whatever the reason for lack of tolerance for the conjunction of time discounting and uncertainty discounting, situations of this type occur quite often in real life. For example, stock exchange investment decisions involve uncertain and delayed payoffs. An investor buying stocks does not know the future value of purchased stocks. Likewise, a farmer does not know the future price of corn. Thus, they may take a risk or consider whether to buy an option, signing an agreement with a company purchasing corn in order to fix a price for it. Also, an owner of land exposed to flooding may consider buying insurance or take a risk of future loss, etc. There are numerous anecdotal observations that people do not like complex risks consisting of delay and uncertainty. Thaler and Benartzi (2007) noted that people prefer bond pension funds rather than stock funds despite the fact that, while more risky, the latter bring a larger rate of return in the long-term. Likewise, when considering whether one should accept an instant indemnity proposed by a defendant or wait for a higher but uncertain indemnity obtainable at court, people are inclined to accept the instant smaller indemnity rather than wait for the higher but uncertain indemnity. The present results suggest that people do not tolerate a delayed risk when they evaluate delayed lotteries in terms of their present value. For example, an investor hesitating over whether to buy a stock fund or a bond fund may take a different decision depending on the type of framing imposed on them. If we make them compare the value of their present money with an uncertain future value of a stock fund, then they may want to avoid the complex risk consisting of delay and uncertainty and therefore reject the stock fund in favor of the bond fund which is supposed to be stable and predictable. On the other hand, if we make them compare

a certain smaller future value of a bond fund and an uncertain, but much higher, future value of a stock fund, they may choose the stock fund.

Our results may be interpreted as indicating that risk attitudes are unstable. In this sense they are in agreement with many previous findings showing that measurement of human risk attitude is not a trivial task. Indeed, Warneryd (1996) showed that various risk assessment techniques and risk attitude measures are poorly correlated. For example, when a participant is asked to choose between a lottery and a certain payoff of the same expected value, their risk attitude may be different than when they are asked to choose between two lotteries of the same expected value. Also, Lichtenstein and Slovic (1971) observed a preference reversal phenomenon: when participants are asked to evaluate lotteries in terms of their certainty equivalents they tend to value lotteries with a high chance of winning a small prize less lower than lotteries with a small chance of winning a large prize. When, on the other hand, they are asked to indicate in direct comparison which lottery they prefer, they frequently prefer lotteries with a high chance of winning a small prize over lotteries with a small chance of winning a large prize. In turn, our experiment showed that risk attitude may change when it is measured using different reference points (a delayed vs. an instant reference point). This may be because people are not highly familiar with the notions of "risk" and "chance". People do not deal with the whole spectrum of probabilities. Besides "certain" and "impossible" they use terms such as "likely" and "possible". Furthermore, studies show that, when facing risky decisions, people are quite often not interested in receiving information about probabilities (Huber et al., 1997; Tyszka & Zaleśkiewicz, 2006).

Finally, our results supported the third hypothesis that temporal distance makes probability weighting curves more linear (people are more sensitive to changes in probability). This result seems to be in line with the finding that temporal distance has an impact on people's feelings, with immediately available rewards evoking stronger emotions than rewards available in the future (Abdellaoui et al., 2011; Sagristano et al., 2002; Savadori & Mittone, 2015). One could therefore assume that people are more calm in their decision making when lotteries are more distant in time. Indeed, McClure et al. (2004) showed that decisions involving immediately available rewards activate parts of the limbic system associated with the midbrain dopamine system, including the paralimbic cortex. Activation of these emotion-related parts of the brain was significantly lower during the making of decisions involving rewards only available in the future. In turn, as shown by Rottenstreich and Hsee (2001), lotteries containing more emotional events lead to less linear probability weighting curves than lotteries containing fewer emotional events and, indeed, our probability weighting functions were more linear for delayed than for instant lotteries.

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