CAN CONJUGATE PRIOR PROBABILITY EXPLAIN THE ILLUSION OF CONTROL?

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Abstract: *In this paper, we consider the illusion of control by using Bayesian updating as the rationality model. Our paper contributes twofold. First, we empirically verify that the illusion of control may have two concurrent sources, "emotional" and "rational". The first one produces biased Bayesian processing due to emotional engagement and the second one yields biases due to prior assumptions on the level of control. Second, we propose a method for identifying these two sources. Moreover we verified two hypotheses H1: The emotional factor*

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causes overestimation of the actual level of control. and H2: The rational factor is responsible for the reverse relationship between observed levels of the illusion of control in three separate situations, when subjects have significant control, moderate or no control. Only the hypothesis H2 received partial empirical support.

Key words: *overconfidence; illusion of control; emotional and rational model; Bayesian updating; conjugate prior probability.*

Classification codes:

D81 Criteria for Decision-Making under Risk and Uncertainty

D83 Search • Learning • Information and Knowledge • Communication • Belief • Unawareness

C53 Forecasting and Prediction Methods • Simulation Methods

CZY WNIOSKOWANIE BAYESA I ROZKŁADY SPRZĘŻONE MOGĄ TŁUMACZYĆ ZJAWISKO ILUZJI KONTROLI

Streszczenie: *W niniejszym artykule rozważamy zjawisko iluzji kontroli, stosując wnioskowanie bayesowskie, jako model racjonalnego podejmowania decyzji. Nasz artykuł ma dwojaki wkład. Po pierwsze, sprawdziliśmy empirycznie, czy iluzja kontroli może mieć dwa równoczesne źródła: „czynniki emocjonalne" i „czynniki racjonalne". Pierwszy z nich wynika z błędów we wnioskowaniu bayesowskim z powodu emocjonalnego zaangażowania, a drugi – z wcześniejszych założeń dotyczących poziomu kontroli. Po drugie, proponujemy metodę identyfikacji tych dwóch źródeł. Ponadto postawiliśmy i zweryfikowaliśmy empirycznie dwie hipotezy H1: Czynnik emocjonalny powoduje zawyżenie faktycznego poziomu kontroli oraz H2: Czynnik racjonalny jest odpowiedzialny za odwrotną relację między oszacowaniami bezwzględnego poziomu iluzji kontroli w trzech oddzielnych sytuacjach, gdy podmiot ma znaczną kontrolę, umiarkowaną kontrolę lub brak kontroli nad uzyskiwanymi wynikami eksperymentu. Tylko hipoteza H2 otrzymała częściowe wsparcie empiryczne.*

Słowa kluczowe: *nadmierna pewność siebie;, iluzja kontroli, emocjonalny i racjonalny model, wnioskowanie bayesowskie, rozkłady sprzężone.*

1. Introduction

Aim of the study

The goal of this paper is to investigate and find the probabilistic foundations of the illusion of control phenomenon, and to examine the illusion of control in a range of control levels, not only in the conditions of no real control and total control, but also for negative control when the objective probability of the desired outcome is decreased by subjects' involvement.

Illusion of control was defined as *an expectancy of a personal success probability inappropriately higher than the objective probability would warrant* (Langer, 1975, p. 313). An excellent illustration was provided by Luo (2004) in the New York Times article about pedestrian traffic lights. New Yorkers pressed the button, believing it would accelerate the appearance of the light that allowed them to cross the street. However pressing the button actually had no impact at all on the traffic lights (see also Gino, Sharek & Moore, 2011). The natural measure of the illusion of control is the difference between the perception of the own impact on the process of generating results and the objective influence on the results. This approach corresponds to the definition of the illusion of control given by Langer (1975, p. 313), where we compare *an expectancy of a personal success probability* and *the objective probability*. Gino et al. proposed a *measure for perceived success of the efficacious action* as the difference between (a) the percentage of successes (the time the blue circle appeared on the screen) when the participant had undertaken some action (after she/he pressed the button) and (b) the percentage of successes when no action was undertaken (when she/he did not push the button, see. Gino et al., 2011, p.110. Due to the illusion of control, subjects think that they can increase the chance of success if they are more involved in the random process. They believe in higher chances of success if they can select the winning numbers in a lottery game or directly roll a die by themselves, (Dunn & Wilson, 1990; Langer, 1975).

There is a dispute in the literature if the illusion of the control is a part of the broader phenomenon of overconfidence. Several manifestations of overconfidence have been identified (overestimation of one's actual ability, performance, level of control, or chance of success), overplacement (people believe themselves to be better than others), and excessive certainty regarding the accuracy of one's beliefs (overprecision). Thus illusion of the control is a special case of overestimation, where perceived control, rather than ability, is overestimated (Presson & Benassi, 1996). There is no agreement as to whether these propensities are manifestations of one (Griffin & Brenner, 2004, Burson, Larrick, & Klayman, 2006; Larrick, Burson, & Soll

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2007) or several different psychological phenomena (Moore, 2007; Moore & Healy, 2008; Glaser & Weber, 2007).

Traditionally, the research paradigm was focused on the situation when the subjects have no (or little) real control over the outcomes of the random phenomenon and are asked about their perceived control. Usually the perceived control was higher than the real control, people overestimated their control, and this resulted in the existence of the illusion of control. This approach impedes the identification of control underestimation, since it is impossible to underestimate the value that equals zero. Some researchers were focused not only on uncontrollable tasks with zero real control, but also on controllable tasks with positive control. By positive control, we mean the situation when the objective probability of the desired outcome is increased by subjects' involvement. Different levels of the real control provide the possibility for making research on the illusion of control when both over – and underestimation of real control is possible. Alloy & Abramson (1979), in their "button-light" experiment, provided such a research schema that enabled subjects to either underestimate or overestimate their real control. Subjects were presented the sequence of yellows lights and interchangeably, in a random way, green or red lights. The subjects had an option of pressing or not pressing a button within three seconds after the yellow light came on. The experimental conditions varied the frequency that the green light came on after the subject pressed the button or did not press the button. Each subject was given 40 trials of the task. After this was done, each subject was given a printed Judgment of Control scale that ranged from 0 to 100 and asked to indicate the amount of control they had over the onset of the green light. The results showed that subjects tend to underestimate their control when it is high and overestimate it when it is low. A similar experimental design was used by Gino et al. (2011). They stated that people underestimate their real control when they have it, but overestimate it when they do not have real control.

Gino et al. (2011) also posed the problem: *how exactly regressive judgments might arise in illusion of control* with the suggestion for further research: *We believe that regressive judgments result from the following simple model: Observed frequency of an event (which may be close to the true frequency of the event) plus error plus prior beliefs. In this model, the error term would incorporate both psychological factors and random noise* (p 112).

Introductory example

Let us assume, as an simplified example that the participant takes part in an experiment, that consist of two tasks. In the first task the participant observes a series of 5 random results and each of them can either be a success or a failure. The participant knows that the results are statistically independent and that the unknown success probability is constant. Moreover the success probability is controlled by steering in each of the five observed experiment results. In such a situation, we say that a participant is involved. The value of the real and unknown success probability is 0.75.

An illustrative example of the Bayesian updating process in presented in Figure 1. The participant believes about unknown success probability can be mathematically represented as a density function of the random variable over possible success probabilities. E.g. participant may think that with the probability 90% the probability of a success in the experiment is lower than 50%. First the participant start with uninformative prior (she/he knows nothing) about potential probability value (top left graph, with uniform distribution). Moreover participant assumes that each value from the range [0,1] is equally probably. Having observed the first success the participant updates her/his beliefs about the unknown success probability, Now the participant presumes that it is more likely that the success probability has rather a higher than lower values (a posteriori distribution presented in the top middle graph). The updating process can be described more formally by the Bayesian formula:

$$P(\theta|D) = \frac{P(D|\theta) \times P(\theta)}{P(D)} \tag{1}$$

Where θ is unknown success probability, $P(\theta)$ represents a prior distribution (equals 1 – non informative prior – in a first updating step), $P(\mathrm{D}|\theta)$ is the conditional probability of observing the data (success or failure in our case) for a specific value of success probability: θ for the observed success and $1 - \theta$ for the observed failure, P(D) represents marginal likelihood of the observed data and $P(\theta|\mathrm{D})$ is a posterior distribution having taken the data evidence into consideration.

Having observed the second success the participant updates her/his beliefs presuming that it is even more likely (than when observed first success) that the success probability has rather a higher than lower values (top middle graph). Later first failure and then two successes are observed. Mathematical representation of the participant beliefs about unknown success probability is presented in the bottom row of the Figure 1.

Having finished the experiment the participant is asked about her/his perceived probability of success. A participant answers 0.7. So the real probability when involved is 0.75, the estimated probability when involved is 0.8 = 4 (number of successes) / 5 (number of trials) and the perceived probability when involved is 0.7.

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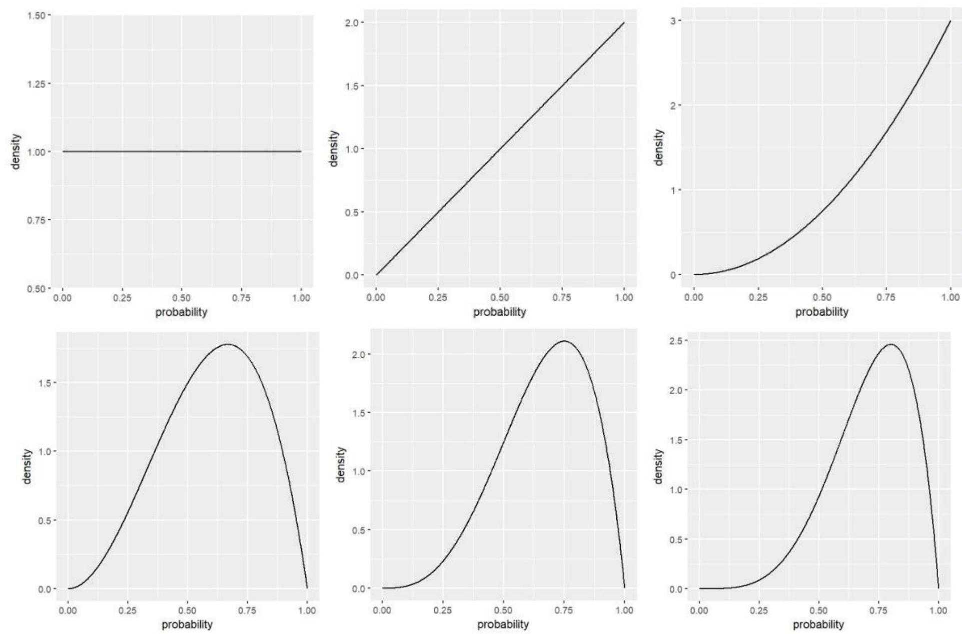

Figure 1. **Example of Bayesian updating process. The participants starts with an uninformative prior (top left graph) and then consecutively updates her/his beliefs observing first success, then second success, then a failure, then third success and as a last the forth success**

The value of the cumulated distribution function of the final posteriori distribution (Figure 2) of the perceived probability (0.7) is 0.42.

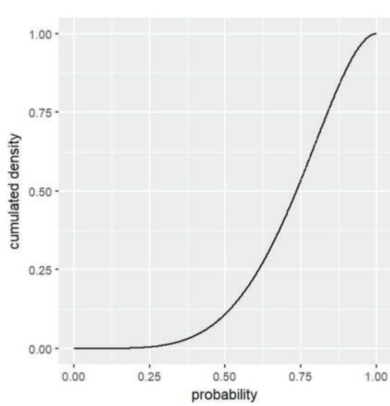

Figure 2. **Example of Bayesian updating process. Cumulated distribution functions representing participant beliefs after the first task of the experiment**

In the second task the participant observes but does not control a series of 5 experiments results. In such a situation we say that a participant is not involved. Analogous results are the real probability when not involved is 0.5, the estimated probability when not involved is 0.4 and the perceived probability when not involved is 0.45.

We may calculate the real control (RC) as the difference between real probabilities when involved and when not involved: RC = 0.75 – 0.5 = 0.25. Analogously the estimated control (EC) equals 0.8 – 0.4 = 0.4 and the perceived control (PC) equals 0.7 – 0.45 = 0.25. In such a case estimated illusion of control (IOC) which is the difference between perceived and estimated control equals 0.25 – 0.4 = –0.15.

Definitions

Formal definitions of the illusion of control and Bayesian updating are introduced in this paragraph. For readers convenience the main symbols used in this and following paragraphs are summarized in the Table 1.

Table 1
Symbols used

Symbol	Meaning
P_I	Real/correct probability when involved
P_N	Real/correct probability when not involved
P_I^P	Perceived probability when involved
P_N^P	Perceived probability when not involved
P_I^E	Estimated probability when involved
P_N^E	Estimated probability when not involved
$P_{\{I,N\}}^{\{.,P,E\}}(i)$	Value of the relevant probability at round i
N_I, N_N	Numbers of trials when involved and when not involved
$N_I^{\nearrow}, N_N^{\nearrow}$	Numbers of successes when involved and when not involved
$B(\alpha,\beta)$	Beta distribution with parameters a and b
$F_I^i(.)$	Cumulative distribution function for posteriori distribution for round i when involved
$F_N^i(.)$	Cumulative distribution function for posteriori distribution for round i when not involved
$\tilde{F}_I^i(.)$	$F_I^i(.)$ with lower and upper tails exchanged for negative real control case
$\tilde{F}_N^i(.)$	$F_N^i(.)$ with lower and upper tails exchanged for negative real control case
$\tilde{F}_I^{1,9}$	Vector of the 9 values of $\tilde{F}_I^i(.)$ at $P_I^P(i)$ for i = 1,…,9
$F_I^{1,9}$	Vector of the 9 values of $F_N^i(.)$ at $P_N^P(i)$ for i = 1,…,9

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In order to formalize the proposed measure of the illusion of control as the difference between the perception of the subject's own impact and the objective influence on the process-generating results, some symbols are now introduced. Actual/correct probabilities are represented by symbols P_I and P_N and estimated/perceived probabilities are assigned to P_I^P, P_N^P, respectively for probabilities while subjects are involved (lower index I) and not involved (lower index N) in the outcomes generating process. The real control is defined by the difference between correct probabilities $RC = P_I - P_N$, while the perceived control is the difference between estimated probabilities $PC = P_I^P - P_N^P$. The measure of the illusion of control is given by the formula[1]:

$$IOC = PC - RC \tag{2}$$

We can state, with reference to real control RC, the conditions of no control ($RC = 0$), positive control ($RC > 0$), or negative control ($RC < 0$), while the illusion of control IOC informs us about underestimating ($IOC < 0$) or overestimating ($IOC > 0$) the actual control. The value of real control RC is a theoretical one. People do not know the exact parameters of the process-generating the outcomes; they can only discover the value of estimators of actual/correct probabilities. Assuming that the outcomes generating process in one step has a Bernoulli distribution and the subject was N_I times involved in that process (for example, by pressing a traffic light button like New York pedestrians), while N_N times she/he was only observing outcomes without being involved, we obtain the binomial distribution. Then estimators of correct probabilities (P_I^E and P_N^E) are given by frequencies:

$$P_I^E = \frac{N_I^{\nearrow}}{N_I} \tag{3}$$

and

$$P_N^E = \frac{N_N^{\nearrow}}{N_N}, \tag{4}$$

where $N_I^{\nearrow}$ and $N_N^{\nearrow}$ stand for the number of successes when the subjects were involved and not involved, respectively. Next, the empirical control is given by the formula $EC = P_I^E - P_N^E$, and then the estimate of the illusion of control is going to be $IOC^E = PC - EC$.

To analyze the exact magnitude of changes of probabilities P_I^P, P_N^P in Bayesian inference, the beta distribution has to be considered, that is the conjugate prior probability distribution for the binomial distributions (Raiffa & Schlaifer, 1961;

[1] We apply this formula in the cases of positive or no control, however this is normalized by multiplying by -1 in the case of negative control to have the same interpretation for under- or over-estimation of one's influence on the results.

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McCausland & Marley, 2013; Turner & Van Zandt, 2012). The beta distribution, as a conjugate prior probability distribution, describes the initial knowledge for probability of success and is given by the following probability density function:

$$f(x|s,f) = \frac{x^s(1-x)^f}{B(s+1, f+1)} \tag{5}$$

where s is the number of successes, f is the number of failures and $B(.,.)$ stands for the beta function $B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt$.

On the other hand, if we assume that subjects are trying to specify correct probabilities (P_I and P_N^P), then they should give their estimates (P_I^E and P_N^E) for which the cdf of conjugate prior probability distributions are equal to the mode values $\frac{s}{s+f}$, so the empirical distribution should not be different from the vector of modes.

The proposed model literature background

We postulate that the illusion of control has two sources, one is connected with the involvement effect (Langer 1975, Dunn & Wilson, 1990), which we will call the emotional factor (this factor results from the involvement in the control activity), and the second one is caused by the prior assumption of subjects about their own influence (level of control) on the process, and we refer to this as the rational factor (this factor is independent from the control activity). The emotional factor is linked to *psychological influences*, while the rational factor is linked to *prior beliefs*. We also propose an exact measure of the illusion of control effect based on Bayesian updating of probabilities. By using a conjugate prior probability distribution in the case of the illusion of control, we obtain the normalization process of probabilities transformations for every individual, which allows us analyze the illusion of control phenomena in the entire spectrum of probabilities at the individual level.

We assume that the illusion of control can be explained by the fallacious use of Bayes' rule while estimating perceived level control (Moore & Healy 2008). Many psychological phenomena were explained by incorrect Bayesian updating of prior probability distributions, like the conservative heuristic (Edwards, 1968) or the base rate fallacy (Bar-Hillel 1980), the representativeness heuristic (Kahneman et al., 1982), the prosecutor's fallacy (Thompson and Schumann, 1987) or even the prediction of binary events (Scheibehenne & Studer, 2014). Moore & Healy (2008) proposed a model based on Bayesian belief-updating processes to explain the mechanism of making judgments about people's performances depending on the task difficulty. Within the model, they describe in a unified way the mechanism underlying three overconfidence forms, i.e., overestimation of one's actual performance, overplacement of one's performance relative to others, and excessive precision in one's beliefs. The

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model proposed by Moore & Healy (2008) describes only the direction in which subjects update their beliefs to new information. In our approach, we refer not only to directional changes, but also to the exact magnitude of Bayesian bias in judgments about the probability of success level within the illusion of control effect.

Within the Bayesian approach to the illusion of control, two aspects of underlying distributions are analyzed. The first is related to systematic differences in Bayesian updating of probabilities while people are involved in the outcomes generating process and when they are not (Dunn & Wilson, 1990); we call this the emotional factor. Within this factor, we suggest that people exaggerate their own impact on the process. The second factor is connected with systematic differences in updating a posteriori probabilities due to prior assumption about subjects own control level. (Alloy & Abramson 1979, Gino et al. 2011); we call this the rational factor. This division is consistent with dual-process theories (see the overviews by Evans & Stanovich (2013), and Osman (2004)) that emphasize that information is processed in two parallel underlying systems: the experiential system (System1) devoted to intuitive thinking and the rational system (System 2) devoted to analytical thinking.

The general hypothesis states that **the illusion of control can be explained by both prior believes used and systematic biases in Bayesian updating of probabilities.** It can be stated in two operational hypotheses connected with the proposed emotional and rational models[2]. Based on the emotional model, which proposes different Bayesian updating when people are involved and not involved in the outcomes generating process, we can propose Hypothesis H1.

H1: The emotional factor causes overestimation of the actual level of control.

The rational model, which is focused on different Bayesian updating caused by prior assumptions about the level of control, causes differences in the illusion of control level depending on the level of control. The illusion of control depends not only on the directional changes of probabilities, i.e., if they are positive or negative, but also on the magnitude of those changes with respect to the assumed level of prior control. This leads us to the second hypothesis:

H2: The rational factor is responsible for the reverse relationship between observed levels of the illusion of control in three separate situations, when subjects have significant control moderate or no control.

[2] The presented approach to analyze the illusion of control by means of posteriori distributions when subjects are involved in the process or not and when they have control or not was motivated by the Copula function (Colonius, 2016). The Copula function is a joint distribution that is used to describe the dependence between random variables that are represented by marginal probability distributions. In our reasoning, two a posteriori distributions (when subjects are involved in the process and they are not) play the role of marginal probability distributions, while the a posteriori distributions (when subjects have positive or negative control or they do not have control at all) reveal dependence between marginal variables.

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As in the case of System 1 and System 2 for information processing (Osman, 2004), we do not assume that one of the models proposed for the illusion of control is predominant. We assume that the proposed models should be identified at the individual level and the methodology, which is described in the next section, can be used for that purpose.

The proposed model general idea and examples

The phenomenon of illusion of control can be represented graphically, see Figure 3. We consider three different real control levels (no real control, positive real control and negative real control). For each control level we consider two situations: without involvement (the participant was not involved in results generation, she/he did not steer) and with involvement (the participant was involved, she/he did steer). If the process of steering has real effect on the experiment result was dependant on the level of real control. In case of no real control participants efforts put in steering had no real effect, in case of positive control steering increased probability of success, while in case of negative control decreased the success probability. We illustrated the cases when participant could correctly guess real probabilities when not involved but overestimated the impact of the steering process (her/his control). Perceived probabilities were higher in case of no or positive control, respectively lower in case of negative control than the real/estimated probabilities.

Exemplary distributions for three different levels of real control are presented in Figure 3.

If we assume that the effect of emotional factor is bigger than the magnitude of error in the case of no involvement, then in the cases of no real control and of positive control, we predict positive illusion of control; while in the case of negative real control, there is also negative illusion of control. We propose to measure the illusion of control as the difference between the perception of subject's own impact on the process-generating results and the objective influence on the results. The subject's own impact is estimated by the difference between the assessed probability level when the subject is involved and when he/she is not involved (the difference between the levels at lower and upper panel marked by the red lines). The objective influence is measured by the difference between objective probabilities (the difference between the levels at lower and upper panel marked by the black lines). In the cases of no real control and of positive control, the differences between probabilities have a positive value, so we observe the positive illusion of control due to the fact that the distance between the red lines (subjective impact on the process) is bigger than the distance between the black lines (objective influence). In the case of negative real control, the differences between probabilities have a negative value, which results in a negative

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illusion of control measure. In "emotional" illusion of control, the bias is due to different Bayesian updating when people are involved in the process and when they are not involved, but not due to prior assumptions on the level of control.

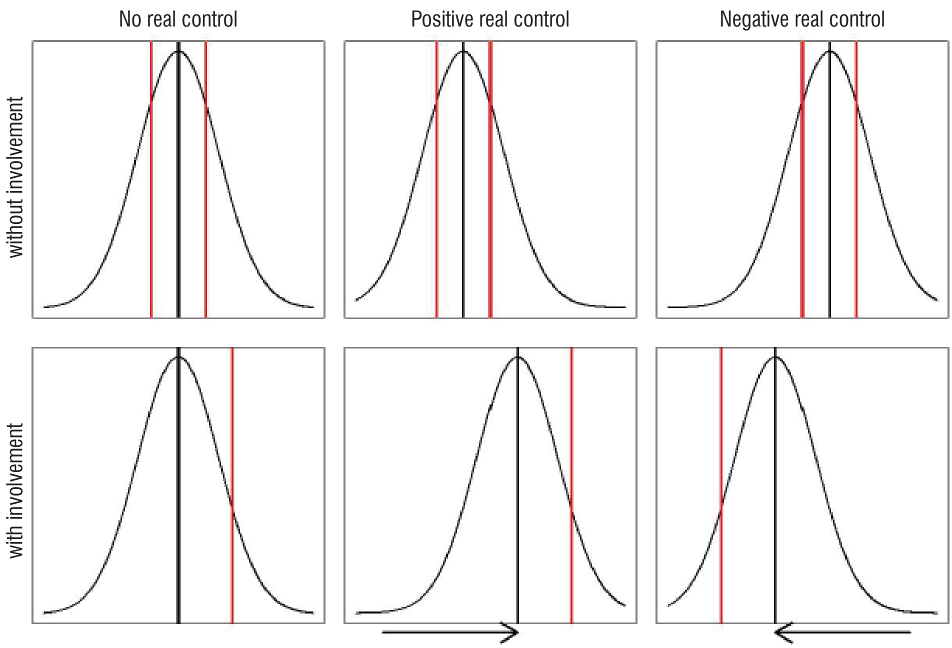

Figure 3. **Cases of exemplary distributions defined by different relative placement of objective and estimated probabilities within the emotional factor. The densities of the posteriori distributions in the Bayesian updating process are represented by the black lines: probability of success are on x axis and the respective densities on y axis, see also Figure 1. The upper graphs present exemplary distributions when subjects are not involved, while the lower graphs present distributions when subjects are involved; the levels of correct probabilities are marked by black lines, while the perceived ones by red lines. The arrows indicate the level of real control**

The second factor influencing a posteriori distributions, called the rational factor, is connected with systematic differences in updating a posteriori probabilities due to the prior assumptions about the level of control when people are involved in the outcomes generating process (Moore & Healy, 2008). We assume that subjects expect to have moderate impact on probabilities of success that is consistent with the direction of objective impact. In the case of no control, subjects previously assumed positive impact and they expect an increased probability level when they are involved. We presume that people can quite accurately estimate the difference between observed probabilities of success when they are involved in the process and when they are not. We assume that people are better at estimating the differences rather than the point values of any physical value. It is analogous to the value function in prospect theory, with the domain defined over the relative changes (gains and losses) rather than final states of wealth (Kahneman & Tversky, 1979). Based on the observed difference between probabilities, subjects modify the prior assumptions about the level of their control. We believe that a prior level of control works as an anchor (Tversky and Kahneman, 1974) and the subject uses this to underweight sample information (Edwards, 1968, Moore & Healy, 2008). Due to this, we can distinguish three scenarios, depending on the relative relationship between the prior and the observed control.

The first scenario takes place if the prior level of control is bigger than the observed one. Subjects will try to assess probabilities of success when they are involved in the process-generating results and they are not involved, in such a way that the difference between their probability estimates will be bigger than the objective difference. This is shown on Panels A, B and E in Figure 4. The distance between the green lines reflects the prior level of control and the distance between the black lines is the objective control, while the distance between red lines is the final perceived control. In the case of no real control (Panel A) and moderate positive control (Panel B), we observe the positive illusion of control level; while in the case of moderate negative control (Panel E), we have the negative illusion of control. The second scenario is when the prior level of control is close to the observed one. Then subjects will make quite accurate estimates of probabilities, making random errors around the objective probabilities. This is the case when the real control is at a moderate level similar to the assumed prior. This is shown on the Panels C and D in Figure 4. In both cases, we do not observe significant values of the illusion of control measure. The third scenario is when the observed control is significantly larger than the prior moderate level of control. In this scenario, due to the smaller prior, subjects will try to assess probabilities of success in such a way that the difference between their estimates will be smaller than the objective one. This is illustrated on Panels E and G. In the case of significant positive real control, we observe negative values of illusion of control (Panel E); while in the case of significant negative real control, we observe positive values of illusion of control (Panel G).

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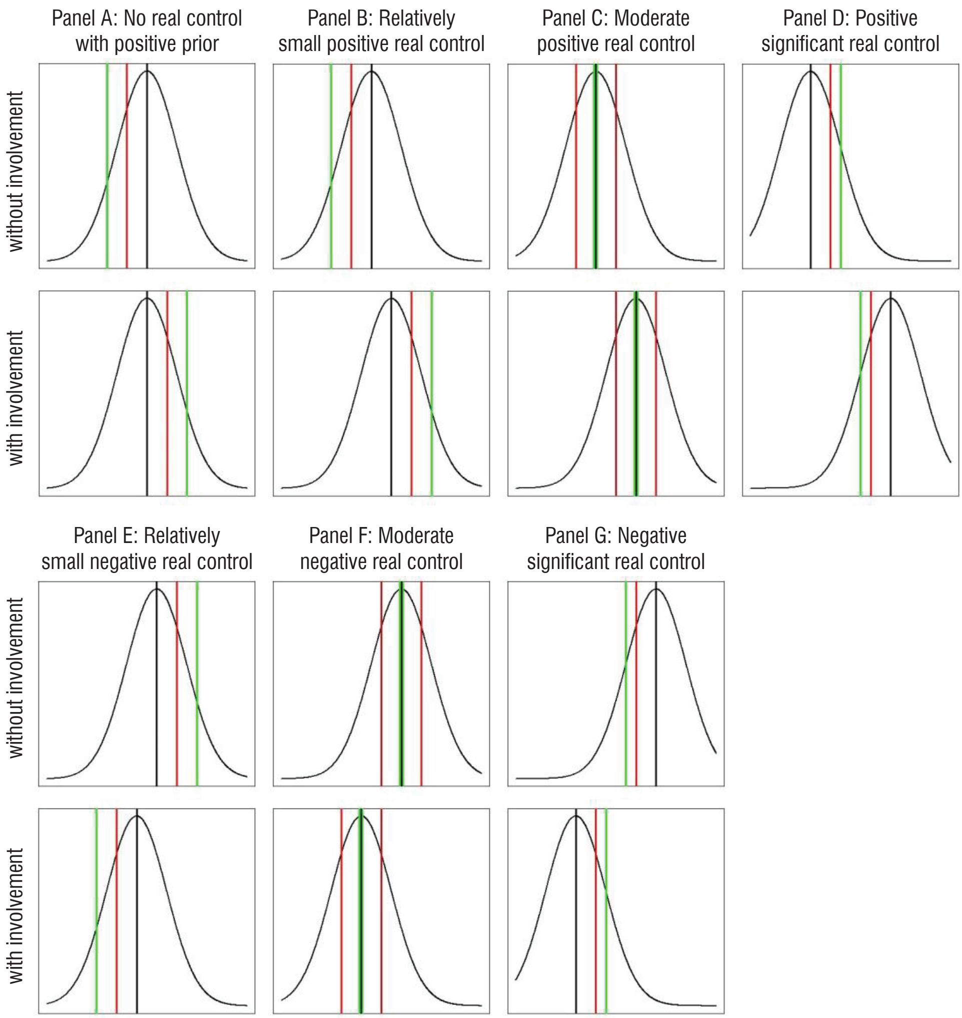

***Figure 4*. Cases of exemplary distributions defined by different relative placement of objective, estimated, and prior probabilities within the rational factor. The upper graphs present exemplary distributions when subjects are not involved, while the lower graphs present distributions when subjects are involved; the levels of correct probabilities are marked by black lines, the perceived/estimated ones by red lines, while the prior levels by green lines.**

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2. Methodology

Participants

Students of the Capital Markets Major of Cracow University of Economics participated in this experiment. The experiment was carried out on a group of 51 (17 women) students of the Capital Markets Major during the Technical Analysis (TA) course. The group was homogenous of 3rd year students with an average age of 22. Participation was voluntary and encouraged by the researcher not associated with TA course teacher. The same independent researcher described a study to participants obtaining informed consent for their participation. The work described has been carried out in accordance with the Declaration of Helsinki for experiments involving humans. Although no monetary incentives were provided, the participants were given bonus credits for the Technical Analysis course and additionally all students with the best results were awarded with extra bonus credits. This was intended to provide higher motivation than any minor monetary payoffs that might have been offered instead.[3]

Design

The Bayesian updating approach to the illusion of control and hypotheses to be tested are presented later in the Introduction section. For the purpose of verifying the stated hypotheses about illusion of control and systematic differences in Bayesian updating of probabilities (which is usually treated in the literature as the benchmark of rational behavior), we have adopted the experiment of Fenton-O'Creevy et al. (2003). Participant observed the simulated path of the stock price on the screen and in selected rounds could additionally influence (steer) the probability of stock price increase in this experiment. In the original experiment (contrary to ours) participants had no real control over the simulated path. This approach allows us to measure the participants' activity in the situations with different levels of control, moreover we also change other parameters like the number of rounds that the participants steer the stock price.

During the study, participants could observe on a screen the simulated price movements (prices were changing stepwise), see Figure 3. The experiment was programmed in Inquisit 4 Lab by Millisecond.

[3] Students receive the monthly scholarship depending on the average grade, so there is a direct relationship between grades and monetary payments. Moreover a good average grade is very important for the 3rd year students as it allows students to avoid taking the entrance exams for MA studies.

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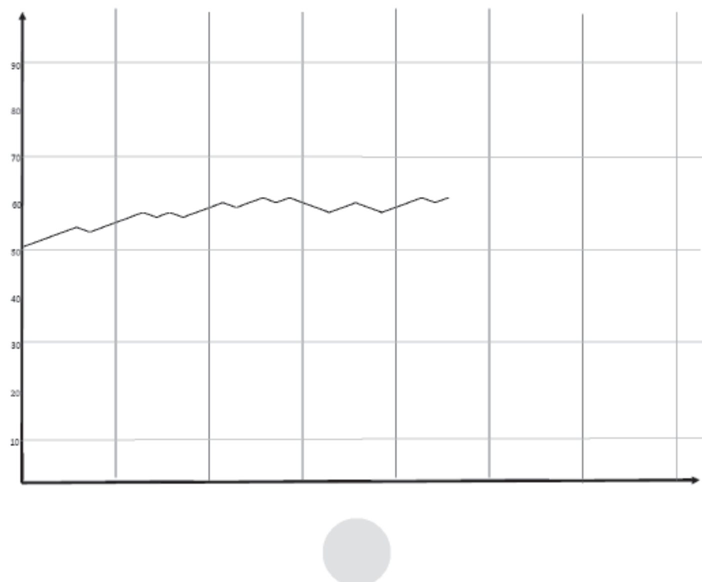

Figure 5. **Print screen of simulated price movements within Round 1 (out of 9)**

Procedure

There are 9 rounds in the study, displayed in the same order for each participant. At the beginning of the study, participants were informed that they would be observing the simulated path of the stock prices on the screen. They were also told that the prices change stepwise in 50 steps, where every step was simulating the time unit. Participants were also told that they would be able to change (however with some[4] delay) the simulated stock prices movements to either little or significant extent by placing the cursor in control field. The control field is the yellow circle at the Figure 5. The participant task was to cause the stock price to reach the highest level in every round by appropriate placing of the cursor over the control field. Participants were also informed about the possibility that their actions could have no impact on simulated prices. At the end of each round of the experiment, participants were asked the questions motivated by Gino et al. (2011) approach:

1. What was the base probability (no steering) of the stock price increase in a single step?
2. What was the probability of the stock price increase in a single step while steering?

[4] no precise definition or measure of delay was given to participants

3. In how many steps had you steered the probability?
4. In how many of these steps had the stock price increased?
5. In how many steps when you had not steered had the stock price increased?

The participant were not allowed to take any notes. However, after each experiment round the whole simulated path remained visible for participants for two minutes. Participants could shorten this period and close the relevant screen earlier.

In each round of the Study 1, we used different probability levels, as in Table 1. Based on the formula for real control $RC = P_I - P_N$, there is no control ($RC = 0$) in rounds: 1, 3, 9; positive control ($RC > 0$) is in rounds: 4, 6, 5 and negative control ($RC < 0$) in rounds: 2, 7, 8. We have moderate (absolute) control in rounds 4,6,and 8 and significant (absolute) control in rounds 2,5, and 7.

Measures and variables

Within the experimental design for every subject and for every round, there are two conjugate prior probability distributions: first when subjects are involved in the outcomes generating process and second when subjects are not involved, with parameters s, f equal to $s = N_I^{\nearrow}$, $f = N_I - N_I^{\nearrow}$ and $s = N_N^{\nearrow}$, $f = N_N - N_N^{\nearrow}$ respectively for with and without involvement in the process. We find cumulative distribution functions (cdf) for two conjugate prior probability distributions and find their values for arguments of the perceived probabilities P_I^P, P_N^P respectively. This procedure should be conducted for every round within the experimental design, for which probabilities of success, when subjects are involved in the outcomes generating process and when subjects are not involved, are fixed. By analyzing the vector of cdf values of perceived probabilities given by subjects for every round, we can infer the accuracy of the subjects' judgments about posteriori probabilities. The vector of cdf values of perceived probabilities plays the role of the empirical distribution[5] and it can be used to check individual tendencies in probability judgments when subjects are involved and not involved in the process of generating outcomes. The values of conjugate prior distributions for perceived probabilities inform whether the subject over- or underestimates the observed actual frequencies, because the beta distributions results are already normalized and we can make inferences about the error terms. When, as in our experiments, people are faced with similar or the same number of experimental rounds with no real control, moderate control, and significant real control, if both empirical distributions are similar, then subjects have the tendency to make similar Bayesian updates (errors) in the cases of steering (involvement) and non-steering

[5] We transform appropriate values in the case of negative control by subtracting the empirical values from 1 to get the same interpretation in the case of positive control. Overestimating the influence when involved, regardless of being negative or positive, leads to positive illusion of control

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(non-involvement). If the empirical distribution in the case of subjects' involvement stochastically dominates the empirical distribution in the case when subjects are not involved, then Hypothesis H1 receives empirical support. As illustrated in Figure 3, we can state that due to the emotional factor, subjects overestimate probabilities when they are involved in the process, which causes the stochastic domination of the empirical distribution in the case when subjects are involved over the empirical distribution in the case when subjects are not involved. If there is a negative correlation between the empirical distributions, then H2 receives empirical support. As illustrated in Figure 4, we see that due to the rational factor, the biases are opposite; i.e., the more subjects overestimate/underestimate probabilities in case of involvement, then the more they underestimate/ overestimate probabilities in case of non-involvement. In the case of Hypothesis H2, it is very important to examine the empirical distributions generated within the experimental design with different levels of real control; i.e. moderate, significant, and no real control.

The emotional and rational models for illusion of control can be examined by means of empirical distributions in the cases when subjects are involved and are not involved, but of course intermediate processing models and other models are possible too.[6] For example, if we assume a non-informative prior, when the subjects are giving random estimates of the probability, then the empirical distribution should not be different from a uniform distribution with support on the [0;1] interval.

To apply the Bayesian approach, as we have stated in the *Exact magnitude of changes in estimating probabilities* section, for each round (i) we found two cumulative distribution functions (cdf) for a posteriori probabilities (beta distribution) for the case when subjects were steering the process $F_I^i(.)$ and not steering $F_N^i(.)$, where the number of steps when the stock price increased s and the number of steps when stock price decreased f were estimated for the given i-th round. Then for perceived probabilities of increasing stock prices in i-th round while steering $P_I^P(i)$ and $P_N^P(i)$ not steering for each student, we got the vector of maximum 9 (1 each for each round) empirical cdf values: $F_I^i(P_I^P(i))$ and $F_N^i(P_N^P(i))$. In order to get consistent results in case of positive and negative control, we used the lower tail cdf in the cases of positive or no real control and the upper tail in the case of negative control; and we mark those results as $\tilde{F}_I^i(P_I^P(i))$ and $\tilde{F}_N^i(P_N^P(i))$ and refer further in the text in the simplified way $\tilde{F}_I^i$ and $\tilde{F}_N^i$. In such a way, the higher empirical values indicate the overestimation of probability compared to the mode of the Bayesian posterior probability. In the case of the no control situation (RC = 0), different approaches could be taken; however we have decided to assume the perceived positive control

6 The same approach is used in multivariate statistics where multivariate distributions are separated into marginal distributions and the correlation structure among them is examined (e.g. by applying the Copula functions).

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indicates overestimation of control, whereas the perceived negative control indicates underestimation of control. To verify the Hypotheses H1 and H2, we analyzed each student empirical distribution over nine rounds, $\tilde{F}_I^{1,9}$ and $\tilde{F}_N^{1,9}$, that are marginal a posteriori distributions of probabilities (of success) while subjects are involved in the outcomes generating process and when they are not involved. As a measure of differences between marginal a posteriori distributions of probabilities while subjects are involved in the outcomes generating process and when they are not involved, we take the cdf-value of a one-sided Kolmogorov-Smirnov test (the symbol *K-S* is assign to this variable). The alternative hypothesis in the Kolmogorov-Smirnov test states that marginal a posteriori distributions of probabilities while subjects are involved in the outcomes generating process stochastically dominates a posteriori distributions while subjects are not involved, so the higher values of *K-S* are then the differences between empirical distributions are more significant.

For every round (i) and for every subject the estimator of the illusion of control value has been found

$$IOC^E(i) = \begin{cases} PC(i) - EC(i); \text{ for no or positive control} \\ EC(i) - PC(i); \text{ for negative control} \end{cases} \tag{6}$$

and then the average value over all 9 rounds was calculated:

$$\overline{IOC} = \frac{\sum_{i=1}^{9} IOC^E(i)}{9} \tag{7}$$

3. Results

Results for individual data within Bayesian approach

First we checked if the participants' empirical distributions, $\tilde{F}_I^{1,9}$ and $\tilde{F}_N^{1,9}$, are marginal a posteriori distributions of probabilities (of success) while subjects are involved in the outcomes generating process and when they are not involved differ significantly from an assumed theoretical uniform distribution. For the no steering case, $\tilde{F}_N^{1,9}$, there were 12 subjects (out of 51) showing significant differences from a uniform distribution at the significance level of 5% in the two-sided Kolmogorov-Smirnov test (R Core Team (2016).)). Then to check the significance of results across all respondents, we ran the binomial test for multiple tests, and the p-value is significant ($p < .0001$). For the steering case probabilities $\tilde{F}_I^{1,9}$, we found significant differences from a uniform distribution for 22 subjects (out of 51) and also a statistically significant p-value for the binomial test ($p < .0001$).

Next we compared empirical distributions with mode values of conjugate prior probability distributions. For the non-steering case probabilities, $\tilde{F}_N^{1,9}$, there were 15 (of 51 students) showing a significant difference ($p < .05$) in Kolmogorov-Smirnov two-sided test and the binomial test for multiple tests gives a significant result ($p < .0001$). For the steering case probabilities, $\tilde{F}_I^{1,9}$, there were 18 (of 51 students) showing a significant difference ($p < .05$) in the Kolmogorov-Smirnov two-sided test, with a significant p-value for the binomial test for multiple tests ($p < .0001$).

We can observe that some subjects have a systematic bias in Bayesian updating of posterior probabilities and the number of such subjects is higher in the case of the steering (involvement) situation.

We then directly compared the distributions between empirical cdf for steering $\tilde{F}_I^{1,9}$ and no steering situations $\tilde{F}_I^{1,9}$ for each subject separately. We applied Kolmogorov-Smirnov and Wilcoxon tests (R Core Team (2016)). In some cases due to limited number of observations or ties, only the approximate p-value of the test could be calculated. It occurred that only 10 (12 for Wilcoxon test) subjects have significant differences in Bayesian updating bias in case of steering and no steering situations (p-value of the binomial test for multiple tests are .01 and $p < .0001$, respectively for results of the Kolmogorov-Smirnov and Wilcoxon tests).

We then calculated the correlations between distributions $\tilde{F}_I^{1,9}$ and $\tilde{F}_N^{1,9}$. We excluded the subjects with less than 6 empirical cdfs, thus the results were obtained for 49 subjects. Of these, 26 subjects had negative correlations (with a mean correlation of -.43 and a standard deviation of .19), while 23 subjects showed positive correlations with a mean of .33 and a standard deviation of .21. The overall mean correlation coefficient in the group of subjects is –.07 with a standard deviation of .43.

To identify those participants who made judgments about probabilities being driven by the emotional factor (H1) or the rational one (H2), we formed clusters based on the values of correlation coefficient between empirical cdfs and values of the one-sided Kolmogorov-Smirnov test (K-S). To build the cluster, we first applied hierarchical clustering procedure with Euclidean distance and Ward's method (R Core Team 2016); then the final clusters were chosen by using the dynamic tree-cut procedure with the minimal number of observations set to 8, which is the square root of the total number of observations, (see Langfelder & Horvath, 2008). The clustering results are shown in Table 5.

The IOC variable for three control levels is defined as an average of $\overline{IOC}$ but only for selected rounds with no real control, moderate control, or significant control separately. Cluster no. 1 represents students that were driven by the emotional factor, due to significant values of the K-S variable. This group was prone to the illusion of control effect in the cases of no real control and moderate control Cluster no. 5 represents students that were driven by rational factor, due to significant values of correlation coefficients. This group had negative values of the illusion of control measure in the cases of no real control and significant control.

Two groups (1 and 3) with positive illusion differ in the sign of the correlation coefficient and the value of the KS test. Group 3 students ("rational" IOC) strongly overestimate the influence when involved in the case of no control, however they also strongly underestimate the influence when involved in the case of significant control in comparison with the Group 1 students ("emotional" IOC).

The values of perceived control (PC), real control (RC), and empirical control (EC) are presented in Table 3. Perceived control (PC) is the mean difference of the subjects answers to the questions about steering and no steering probabilities after each round, while empirical control (EC) is the mean difference between observed frequencies of increased stock price when steering (P_I^E) and not steering (P_N^E). Real control (RC) is the difference in parameters – base and steering probabilities from Table 2. The number of observations differ in Table 3 due to the first and the last step correction in every round. In the first step, subjects could only observe the simulated stock price, but they could not control it. On the contrary, in the last round, subjects could only control the stock price, but they could not see if it had led to a price increase or decrease due to the delay. We also excluded the observations when subjects had not steered for the whole round. In the case of no real control, i.e. Rounds 1, 3 and 9, we observe that perceived control is smaller than empirical control, but there is only one significant result in the case of Round 1. When we take all rounds with no real control and compare PC with EC, then we find that perceived control is smaller than empirical control, t(272.1) = –2.28, p= .023.

In the case of negative control, i.e., Rounds 2, 7 and 8, there are significant differences in each round between perceived and empirical control levels (for all three rounds together, the t-test values are t(254.7) = 5.50, p < .0001). It is observed that the absolute value of perceived control is smaller than the absolute value of empirical control, which means that subjects underestimate the change of probabilities in the steering condition with respect to the non-steering one. As a result, there is a positive value of illusion of control ($IOC^E = PC - EC$) in the case of negative control. For positive control, there are significant differences between PC and EC in Rounds 4 and 5, while in Round 6 the difference is not significant. For all three rounds together, the perceived control is statistically significantly smaller than the empirical control. In the case of positive control, the value of the probability of increasing the stock value is rising while steering and the subjects have not updated the value of this parameter sufficiently.

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We analyzed how the empirical and perceived probabilities of increasing stock price differed in case of steering (Table 4) and no steering (Table 5) to verify H1. Perceived stock price increase probability (P_I^P and P_N^P) is the subjects' answers to the questions of steering probability at the end of each round, while empirical stock price increase probability (P_I^E and P_N^E) is the actual frequency of growth in every round. There is a common tendency that can be observed in cases of steering (Table 3) and non-steering conditions (Table 4). One aspect is that the perceived probabilities are closer to the empirical ones when they are close to the middle probability value of .5; and the second issue is that they are closer to .5 than the empirical probabilities when the empirical probabilities differ significantly from the middle value of .5. It appears that the respondents over-weighted relatively small probabilities, while they underestimated relatively higher ones. This effect is more clearly observed when subjects are involved in the outcomes generating process; we can observe 6 out of 9 significant differences in Table 4, while only 3 out of 9 in Table 5.

Discussion

The data presented in Table 3 confirms the observations of Gino et al. (2011) in the case of positive control. But there is no positive illusion of control in the case of no real control, which was expected based on the literature review (Langer, 1975; Gino et al., 2011). This can be explained by the students' expertise in modelling stock price behavior by the binomial pricing model in the case where parameters do not change (Cox, Ross & Rubinstein, 1979) and by frequent feedback within the experimental design (Murphy & Winkler, 1984). Hypothesis H1 would receive empirical support if the absolute value of perceived control would be greater than the absolute value of empirical control ($|PC| > |EC|$) and they both would have the same sign. Hypothesis H1 did not received empirical support in any of the nine rounds. This hypothesis needs to be verified on individual level.

The aggregated data for the steering (Table 4) and non-steering conditions (Table 5) partially supports Hypothesis H2. The probability value of 50% serves as an anchor in judgments about probability (Tversky & Kahneman, 1974). As a starting point for the probability estimator, subjects used 50% as an initial anchor, which is then adjusted to accommodate additional information coming from the process realization. Typically the adjustment process is not adequate, resulting in biased values of the perceived probabilities, which are biased towards the anchor of 50% (Edwards 1982).

The results presented in Tables 3, 4 and 5 are based on classical descriptive statistics, i.e., mean and standard deviation for all subjects. They only provide information about the first two moments of the probability distributions across all subjects for every round; they are not sufficient for making conclusions about the differences in distributions of probabilities. The Bayesian approach allows for effective verification of Hypotheses H1 and H2 on an individual level; it allows us to compare Bayesian biases in the cases of steering and no steering.

The small percentage of students with positive IOC and average low level of IOC in the whole group (students on average underestimate their influence when involved) may have two main reasons. First, and the most important factor, is that we applied relatively high values of control (.2 and .4) in 6 of 9 trials when students tend to underestimate their real control. Second, the students that had training in financial markets had a higher than average level of precaution when judging their own influence in this kind of experimental setup.

Based on Bayesian interface and theoretical assumptions about the influence of emotional and rational factors on illusion of control phenomena, we could identify participants that were representative for one or another mode. We observed that both biases, as in Model 1 and Model 2, contribute to the observed illusion of control.

4. General discussion and conclusions

The presented methodology of investigating the illusion of control phenomena based on a Bayesian approach allows us to measure the exact magnitude of biases in estimating a posteriori probabilities. By using beta distributions, which are conjugate prior distributions for realized processes, normalization of probability estimates was achieved. This approach allows the analysis of decisions across the full spectrum of probabilities and for the identification of the illusion of control effect at the individual level.

We verified that the illusion of control may have two different sources that are moderately correlated: "emotional" and "rational" as proposed in the general hypothesis that "the illusion of control can be explained by both prior believes used and systematic biases in Bayesian updating of probabilities". The first one is due to biased Bayesian updating due to emotional engagement; while the second one is due to prior assumptions of moderate control in Bayesian updating. By analyzing the empirical distributions when people are involved and when they are not involved within Bayesian interface, we have verified two hypotheses about the emotional and rational factors causing the illusion of control. We have identified subjects that were driven by the emotional factor, which is connected with traditional understanding of the illusion of control phenomena, namely when people are involved, then they used to overestimate their own impact. In addition, we found a negative correlation between involvement and updating biases, meaning if the subjects underestimate

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the probability when not steering, they tend to overestimate the probability when steering. We call this effect the rational factor. Moreover, we showed that both effects contribute to the observed illusion of control and that illusion of control is an individual trait – the effects can be observed only for part of the student group.

The idea presented in the paper allows to identify two sources of observed illusion of control. This potentially enables to better understand he observed phenomenon of illusion of control and predict it when participants exposed to different stimuli (e.g. partial prior knowledge).

Further research will concentrate on developing the formal decision model that would allow to decompose the illusion of control into two factors related to the "emotional" and "rational" components discussed in the paper.

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Table 2
Base probability (control button released) and steering probability (control button pressed) in Study 1

Round Number	Base probability $P_{N\_I}^{C}$	Steering probability P_{I}^{C}
1	.50	.50
2	.75	.35
3	.25	.25
4	.50	.70
5	.25	.65
6	.75	.95
7	.50	.10
8	.25	.05
9	.75	.75

Table 3
Comparison of perceived, empirical, and real control levels for every round in Study 1

Round Number	Perceived control PC			Empirical control EC			t-test	p-value	Real control RC
	M	SD	N	M	SD	N			
R1	-.04	.23	49	.03	.15	49	-2.103	.041	.0
R2	-.25	.33	47	-.40	.20	47	2.537	.015	-.40
R3	-.05	.16	50	-.02	.17	50	-.986	.329	.0
R4	.07	.30	47	.22	.20	47	-3.158	.003	.20
R5	.13	.29	50	.42	.18	50	-7.646	.001	.40
R6	.18	.34	41	.19	.20	41	-.219	.827	.20
R7	-.22	.23	47	-.42	.11	47	-7.001	.001	-.40
R8	-.11	.15	49	-.19	.11	49	3.813	.001	-.20
R9	-.04	.28	44	.04	.22	44	-1.46	.151	.0

Table 4
Comparison of perceived and empirical stock price increase probabilities when steering

Round Number	Perceived probability when steering P_{I}^{P}			Empirical probability when steering P_{I}^{E}			t-test	p-value
	M	SD	N	M	SD	N		
R1	.41	.20	51	.50	.10	49	-2.874	.005
R2	.37	.24	51	.34	.17	47	.717	.475
R3	.19	.15	51	.23	.09	50	-1.885	.062
R4	.56	.25	51	.68	.09	47	-2.896	.005
R5	.52	.20	51	.66	.08	50	-4.483	.001
R6	.70	.37	51	.94	.04	41	-4.191	.001
R7	.19	.18	51	.08	.07	47	3.861	.001
R8	.08	.12	51	.05	.05	49	1.570	.120
R9	.61	.26	51	.73	.09	47	-2.966	.004

Table 5
Comparison of perceived and empirical stock price increase probabilities when not steering

Round Number	Perceived probability without steering $P^{P}_{N\_I}$			Empirical probability without steering $P^{E}_{N\_I}$			t-test	p-value
	M	SD	N	M	SD	N		
R1	.46	.14	51	.48	.11	51	-.635	.527
R2	.64	.18	51	.74	.10	51	-3.379	.001
R3	.24	.14	51	.25	.14	51	-.236	.814
R4	.52	.19	51	.46	.17	51	1.452	.150
R5	.41	.19	51	.24	.17	51	4.724	.001
R6	.68	.21	51	.76	.18	51	-1.918	.058
R7	.42	.17	51	.50	.08	51	-2.882	.005
R8	.20	.15	51	.25	.10	51	-1.972	.051
R9	.67	.20	51	.70	.19	48	-.750	.455

Table 6
Result of clustering.

Cluster No	No of Observ.	Correlation Coefficient	K-S	IOC^{Total}	*IOC* no control	*IOC* moderate control	*IOC* significant control
1	5	.38	.84	.01	.05	.03	-.04
2	14	.01	.14	-.21	-.13	-.22	-.29
3	12	-.45	.74	.01	.11	.04	-.10
4	10	.47	.08	-.20	-.13	-.24	-.23
5	8	-.61	.15	-.11	-.12	.06	-.29